

పాఠశాల గణితం - దార్శనికత
(Vision for School Mathematics)

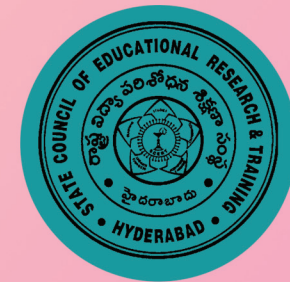
1. పిల్లలు, గణితమంటే భయం పోయి మక్కువతో ఆనందిస్తూ గణితాన్ని అభ్యసిస్తారు.
2. పిల్లలు గణితమనేది సూత్రాలు, యాంత్రిక పద్ధతులకన్నా చాలా గొప్పదని తెలుసుకుంటారు.
3. పిల్లలు గణితాన్ని, భావవ్యక్తీకరణ సాధనంగానూ, మాట్లాడుకోవడానికి, చర్చించుకోవడానికి, కలిసి పనిచేయడానికి ఉపయోగించే ఒక గొప్ప విషయంగానూ చూస్తారు.
4. పిల్లలు అర్థవంతమైన సమస్యలు రూపొందిస్తారు మరియు సాధనలు కనుక్కొంటారు.
5. పిల్లలు, గణితంలో సంబంధాలు కనుక్కోవడానికి, అంతర్గత నిర్మాణాలు తెలుసుకోవడానికి, కార్యాకారణ విచారణకు, ఒక ప్రవచన సత్యాసత్య విలువలు నిర్ధారించడానికి, అమూర్త భావనలను ఉపయోగిస్తారు.

పిల్లలు తార్కికంగా ఆలోచించడానికి, చింతన చేయడానికి, విశ్లేషించడానికి మరియు వ్యక్తీకరించడానికి ఒక వాహనంగా గణితాన్ని చూడాలనేది ప్రత్యేకమైన విషయమే అయినా గణితాన్ని విశ్లేషణ, చింతన అవసరమున్న ఏ ఇతర విషయంతోనైనా సంబంధమున్నదానిగా చూడాలని, జాతీయ విద్యావిధానం 1986 చెప్పింది. జాతీయ విద్యాప్రణాళికా చట్టం NCF-2005 ఈ విషయాన్నే నినదించినా గణిత బోధన ఇంకా సంకుచిత లక్ష్యాల సాధన దిశగానే పోతుండడం విచారకరం.

- రాష్ట్ర విద్యా ప్రణాళిక పరిధి పత్రం - 2011

Handbook for Maths Teachers

Classes 8 & 9

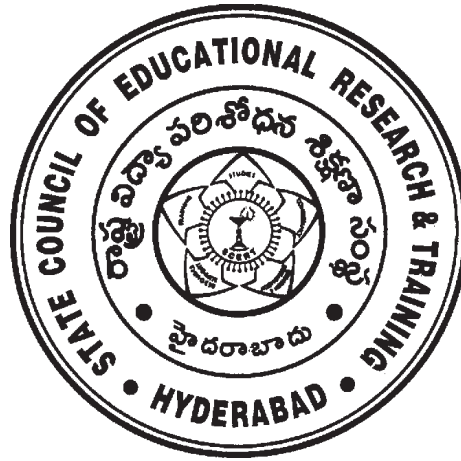


State Council of Educational Research and Training
A.P, Hyderabad.

Hand Book for Maths Teachers

8th, 9th Classes

2013-2014



**State Council for Educational Research
and Training
Andhra Pradesh, Hyderabad**

Module Preparation Team

Sri T.V.Rama Kumar, H.M., Z.P.H.S., Mulumudi, SPS Nellore District.

Sri G.Anantha Reddy, Retd. HM, Ranga Reddy District.

Sri G.V.B.S.N.Raju, S.A., M.H.School, Kasapa, Vizianagaram District.

Sri S.Prasada Babu, PGT, APTWRS, Chandra Sekhara Puram, SPS Nellore District.

Sri P.Suresh Kumar, S.A., GHS, Vijayanagar Colony, Hyderabad District.

Sri P.D.L.Ganapathi Sarma, S.A., GHS, Jamistanpur, Manikeswarnagar, Hyderabad District.

Sri D.Venu, S.A., UPS, Allawada, Chevella (M), Rangareddi District.

Sri Hanif Paliwal, VBERC, Udaipur, Rajasthan.

Subject Experts and Editors

Dr. S.Suresh Babu, Professor, SCERT, A.P., Hyderabad.

Sri K.Narayana Reddy, Lecturer, SCERT, A.P., Hyderabad.

Sri KRajender Reddy, Co-ordinator, Maths Text Books, SCERT, A.P., Hyderabad.

Cheif Subject Expert

Dr. H.K.Diwan, Educational Advisor, VBERC, Udaipur, Rajasthan.

Advisor

Dr. N.Upender Reddy, Professor & Head, Curriculum and Text Book Department,
SCERT, A.P., Hyderabad.

Cheif Advisor

Sri G.Gopala Reddi,
Director, SCERT, A.P., Hyderabad.

Principle Advisor

Smt. V.Usharani, I.A.S.,
Commissioner and Director, School Education,
A.P., Hyderabad.

Foreword

The secondary stage marks the beginning of the transaction from functional mathematics studied upto the upper primary stage to the study of mathematics as a discipline. The logical proofs of propositions, theorems etc., are introduced at this stage. Apart from being a specific subject, it is to be treated as a concomitant to any subject involving analysis as reasoning.

The teacher has to encourage the children to understand and absorb critical issues and abstract concepts duly focusing on teaching learning process instead of scoring marks. It is essential to create a mixed classroom environment for effective transaction of curriculum by involving children participation. Nurturing classroom culture to inculcate positive interest among children with difference in opinions and presumptions of the life style. The new text books make an attempt to concretize all the sentiments. So the teacher has a role and responsibility to create such an atmosphere with skill full thoughts.

To develop such skills among teachers in teaching learning process at different levels preparation, planning and evaluation, the hand book will guide them in as a useful instrument.

This Hand book contains seven chapters in which 1 to 3 chapters give an idea about nature and scope of text book chapters.

Chapter 4 deals precisely with the new concepts introduced in the syllabus. Chapter 5 describes the importance and conceptual understanding of different areas of mathematics to promote teachers to understand deeply in the concerned area. Chapter 6 gives an idea about how to use the new text book in an effective way in teaching learning process such as role of teachers, preparation etc. Finally chapter 7 deals with how to implement continuous comprehensive evaluation in a successful manner.

The state council for Education research and Training appreciates the work done by the committee members, teachers and other technical staff in bringing the hand book in a precise and attractive manner for the benefit of mathematics teachers in the state. In the endeavor to continuously improve the quality of our work with respect to text books and hand books, we welcome the comments and suggestions in this regard.

Director,
SCERT, A.P., Hyderabad.

Index

Contents	Page No.
I. Chapter :	
♦ What is the need of training on new text books?	5
II. Chapter :	
♦ Introduction of New text books - key topics	7
III. Chapter :	
♦ Analysis of a chapter in 8th class text book (Ratio - proportion)	22
IV. Chapter :	
Analysis on Specific topics	
1. Verbal Problems	27
2. Probability	31
3. Proofs in Mathematics	37
V. Chapter :	
Position papers in learning	
1. Number System	44
2. Algebra	51
3. Geometry	59
4. Statistics` probability	63
5. Mensuration	69
VI. Chapter :	
♦ Teacher Preparation	74
VII. Chapter :	
♦ Continuous Comprehensive Evaluation	79
VIII. Appendix :	
♦ Self Evaluation Sheet	96

Chapter 1

What is the need of training on New text books?

Secondary education is undergoing changes time to time. These are necessary as the knowledge widens and the needs of the society from teacher centered education to child centered education. But text books were not changed accordingly it becomes a dream until 2011-12.

Our state Government developed Andhra Pradesh state curriculum frame work (APSCF-2011) in accordance with NCF - 2005 and RTE - 2009. Which recommends that children's life at schools must be linked to their life outside the school. Thus it is mandatory to develop new syllabus and text books involving the children in participate, discuss and take an active part in the classroom processes includes project works, experiments, Analysis. Thus the new text books from class 1 to 10th were developed in a phased manner with a national perspective to prepare our students with a strong base of mathematics and science. Teaching learning strategies were given in accordance with new curriculum. In this occasion, the teachers have to revise their teaching strategies to fit into the new processes. So it is necessary that all the teacher have to undergo an in service training to understand the principles behind developing new text books and teaching strategies.

All the teachers in the state were undergone training last year on 6th and 7th class new text books. The survey conducted by SCERT in the month of Feb213 on "Implementation of New text books - observation of teaching learning processes" throughout the state does not reflect the positive outcome on training. Let us find some conclusion arrived at the end of survey.

Though most of the teachers said that the text books are good, exercises are good and concepts were introduced in a nice manner, but there were some errors observed by the surveyors.

- ◆ There were very less percentage of teachers read "Fore word", "teacher instructions" and academic standards given in the text books
- ◆ Only 10% of teachers are creating new situations in providing activities, discussions, text books reading in the classroom.
- ◆ Very few teachers were given importance to exercise like "Do this", "Try these" and "Think discuss" for better understanding the mathematical concepts.
- ◆ Only few teachers were providing opportunity for children participation in teaching learning process.
- ◆ Very less teachers are providing and working out projects.
- ◆ The understanding of continuous comprehensive evaluation among teachers is very less.

Thus the training programme is meant to provide opportunity to the teachers for better understanding of key principles behind the text books of class 8th and 9th.

Objectives of the training programme :

- ◆ To understand the philosophy and key principles in the new text books of 8th and 9th classes.
- ◆ To discuss and understand the nature and scope of various areas in mathematics.
- ◆ To understand the new and innovative strategies to develop lesson plans and activities.
- ◆ To understand the importance of constructive exercises such as Dothis, Try these and think-discuss.
- ◆ To know how to conduct activities in a maths class.
- ◆ To understand the spirit and assessment techniques through CCE.
- ◆ To prepare the text items based on academic standards.
- ◆ To prepare summative assessment question paper based on weightage tables.
- ◆ To understand about academic calendar and its implementation.

To narrow aim of school mathematics is to develop 'useful' capabilities, particularly those related to numeracy - numbers, number operations, measurements, decimals and percentages, The higher aim includes developing the child's resources to think and reason mathematically, to pursue assumptions to their logical conclusion and to handle abstraction. It includes a way of doing things, and ability and the attitude to formulate and solve problems

- NCF 2005

Chapter - 2

Introduction to New text books - Key principles

Introduction :

Education is a process of human enlightenment and empowerment. Recognizing the enormous potential of education, all progressive societies have committed themselves to the universalization of elementary education with an explicit aim of providing quality education to all. In this connection all the states of our country developed their own curricular keeping in view their self reliance under national perspective. Right to education (RTE - 2009) perceives that every child who enters the school should acquire the necessary skills prescribed at each level upto the age of 14 years. Our State Government has decided to revise the curriculum of all the subjects based on Andhra Pradesh state curriculum frame work (APSCF - 2011) and released 18 position papers on curricular and co-curricular subjects.

The state curriculum frame work 2011 emphasised to utilize the natural learning abilities of the children by creating opportunities in text books to think and apply their knowledge in widening beyond the text books. The exercises in the text books will provide children ample scope to reach the academic standards prescribed in each level.

The new text books were developed by SCERT for class 1, 2 in the year 2011-12, 3,6,7 classes in the year 2012-13 and 4,5,8,9, classes in 2013-14.

To understand the philosophical aspects of these new text books, let us think and discuss the following questions.

1. What are the key principles in the development of new text books?
2. What are the special features of new text books?
3. How the topics are selected for 8th and 9th classes?
4. Which topics are selected were given more importance in the text books
5. How the chapters are arranged in the text books?
6. What are the issues discussed in Fore word and preface of these Maths new text books.

♦ **Key principles in the development of new text books:**

The natural
learning abilities of
the children

Children language
and experiences

Link between topics
with outside
classroom life

Shifting from bi
heating to
knowledge
construction

Topics related not
only related to text
books

Discoveries
through projects

Continuous
comprehensive
evaluation

Sociological and
constructive
approaches

Freedom of
expressions and
questioning ability

Defined
academic
standards

Scope for creativity,
intention

Different ways of
problem solving
approaches

Sl.no.	Topic	Need	Explanation in the Textbooks
1	Simple mathematical language to promote self learning ability	Children feel better to understand the topics in mathematical languages if these topics are linked with home language. If they can able to read and understand the activities and other topics it leads to self learning.	The language used in topics of the 8th and 9th maths books is very simple and understandable by all regions, most of the unfamiliar words were removed. More explanations given where there is a need to introduce new mathematical terminology
2	Activities related to previous knowledge of the child	To motivate the children towards learning a new topic, it is necessary to test them by question related to their previous knowledge through daily life experiences and situations Lines and angles - Homes, bridges temples etc and their figures	8th class 1. Rational numbers - Purchase of pens in a shop - Temperatures in simla 2. Construction of quadrilaterals - Types of quadrilaterals, then properties 3. Frequency distribution tables and graphs - Medals tally in Olympic games - House sites and shapes 4. Area of plane figures 9th class 1. Polynomials - Plantation in a garden 2. సరళరేఖలు, కోణాలు - ఇళ్లు, వంతెనలు, గోపురాలు మొ॥ వాటి పటాలు.

Sl.no.	Topic	Need	Explanation in the Textbooks
3	Giving importance to local art forms, culture, productive works, local needs in lessons	Every region has its own culture, art forms and people participation in useful productive works according to their needs. If these are incorporated in the lessons then children's problem solving abilities will increase and hence it leads to discover new problems	<p>8th class</p> <p>1. Linear equation in one variable</p> <ul style="list-style-type: none"> - Exercise 2.2, problems 6,7,14 relates to models of transport, preparation of pizza, valued the Indian currency (page 43) <p>2. Comparing quantities with proposition</p> <ul style="list-style-type: none"> - Collecting donations in percentage (P.99), value added tax (VAT) medical bill (P.109) etc <p>9th class</p> <p>1. Coordinate geometry</p> <ul style="list-style-type: none"> - order students in introduction and streets in a town (P. 107, 18) <p>2. Surface areas and volumes - volume of boxes, corn cop example if\n finding the surface area of cone etc.</p>

Sl.no.	Topic	Need	Explanation in the Textbooks
4	Academic standards through process skills using problem solving, logical reasoning, communication representation, connections.	Academic standards in maths will be stated combining with content areas and process skills. Academic standard is a statement leads to an output related with content with a process skill. Academic standards will helps us to assess the children	<p>All the chapters and contents in 8th and 9th class mathematics were developed by taking academic standards into consideration</p> <p>Eg:- Problem solving - 8th class(P.17)</p> <p>1) Simplify $\frac{2}{5} + \frac{3}{7} + \frac{6}{5} + \frac{13}{7}$</p> <p>2) Construct an isosceles triangle given box and box angle write its proof communication - 8th class (P.94).</p> <p>3) Express 32.5 x 10 in general form Representation - 9th class (P.136)</p> <p>4) Draw the graph of $x - 2$, $y = 3$ connection - 9th class (P.258)</p> <p>5) A villager Ramayya has a plot of land in the shape of a quadrilateral. The grampanchayat of the village decided to take over some portion of his plot from one of the corners to construct a school. Ramayya agrees to the above proposal with the condition that he should be given equal amount of land in exchange of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.</p>

Sl.no.	Topic	Need	Explanation in the Textbooks
5	Continuous comprehensive evaluation through "Do this", "Try these" and Think-discuss and write	Questions gives under the title 'Do this' will help the teacher to assess the children about their immediate understanding of the topics they have learning. The questions will also help the children for self assessment. The questions under "try these", "think - discuss and write" will help the children to give answer through reasons and discussing in groups.	<p>Question for</p> <p>Do this : Immediate responses about understanding the topics learned.</p> <p>Eg: Finding the medium of height of cricket players 8th class - Frequency tables and graphics</p> <p>Try these : Question related to reason and topic.</p> <p>Eg: Rehan said there are 37 non perfect square numbers between 92 and 112 Is it true? Give reason 8th class squares and square roots (P.128)</p> <p>Think - Discuss and write :</p> <p>Eg: How the compound interest change if interest is calculated for every three months? How many periods will be there in one year? What is the percent of rated interest of 3 months with annual interest?</p> <p>Discuss with your friends 8th class - Appreciation with ration and proposition</p>

Sl.no.	Topic	Need	Explanation in the Textbooks
6	Shifting from bi hearing to knowledge construction	Education should be used to construct knowledge rather than gaining. The structure of topics should help in this direction. Children should be able to discover new things their self experiences matching with topics they learn in the class rooms. It will avoid role learning of a topic or a concept.	Many topics were introduced in new text books which promote knowledge construction Eg: 1. Constructing quadrilaterals - 8th class Activities were given to find the measured to construct a unique quadrilateral (P. 59,60) 2. Statistics - 9th class Definitions were introduced by associating real life situations with 'data' (P.194,195)
7	Developing freedom of	Construction of knowledge among children is possible only when they question themselves about every topic as how? Why? They have to question the book, teachers and their friends. The teacher has to create such an atmosphere in the classroom. The society should create this outside the classroom. Answer many differ but the conclusion should be carried only through proof on teaching of the topics	More situations were provided in the new text books in topics and exercises to develop questioning ability Eg: 8th class - Exponents and power exercise 4.2 question 5 (P.95) 9th class - "quadrilaterals" discussion on about "All squares are rhombuses" (P.176)

Sl.no.	Topic	Need	Explanation in the Textbooks
8	Vie variety of examples to understand problem solving techniques and exercise to solve with joy.	Scope should given to children to solve problems on their own. So it is necessary to give more examples to understand the steps and logical process in which a problem can be solved. Every teacher must make the students to understand the problem solving techniques. It removes the fear to solve problems given in the exercises. Children must leads to create their problems by their own logic related to the given concepts	More number of examples were given rather than problems in the exercise in the new text books Eg: 1. "Square roots - cube roots" (8th class) contain 5 exercise and 15 examples, not more than 10 problems were given in every exercise 2. Triangles (9th class) consist of 4 exercises and 15 examples
9	Geometrical construction with logic and reasoning	Geometrical constructions play a vital role in ancient times in the study of mathematics. The principles followed in the constructions were developed since enchild's period. Every geometrical construction involves not only beauty but topic construction were said to be accurate and standard only. When we use ruler and compass, children must adopt to use them in school days only will help them in engineering level.	Two chapters "construction of quadrilaterals" (8th class) and "Geometrical constructions" (9th class) provide ample scope for the children to construct geometrical figures using ruler and scale. The geometrical proofs of these constructions will help them to understand the logic behind the constructions. Eg: construction 3.2.1 (P:65,66) - 8th class, construction 13.3.1 (P:284,285) 9th class

New Mathematics Text Books Special Features

a) Cover Pages



b) Foreword

8th Class

Foreword

Education is a process of human enlightenment and empowerment. Recognizing the enormous potential of education, all progressive societies have committed to the Universalization of Elementary Education with an explicit aim of providing quality education to all. As the next step, universalization of Secondary Education has gained momentum.

The secondary stage marks the beginning of the transition from functional mathematics studied upto the upper primary stage to the study of mathematics as a discipline. The logical proofs of propositions, theorems etc. are introduced at this stage. Apart from being a specific subject, it is to be treated as a concomitant to any subject involving analysis as reasoning.

I am confident that the children in our state of Andhra Pradesh learn to enjoy mathematics, make mathematics a part of their life experience, pose and solve meaningful problems, understand the basic structure of mathematics by reading this text book.

For teachers, to understand and absorb critical issues on curricular and pedagogic perspectives daily focusing on learning rather than of marks, is the need of the hour. Also coping with a mixed class room environment is essentially required for effective transaction of curriculum in teaching learning process. Nurturing class room culture to inculcate positive interest among children with difference in opinions and presumptions of life style, to infuse life into knowledge is a thrust in the teaching job.

The afore said vision of mathematics teaching presented in Andhra Pradesh State Curriculum Framework work (APSCF - 2011) has been elaborated in its mathematics position paper which clearly lays down the academic standards of mathematics teaching in the state. The text books make an attempt to concretize all the sentiments.

The State Council for Education Research and Training Andhra Pradesh appreciates the hard work of the text book development committee and several teachers from all over the state who have contributed to the development of this text book at different levels. I am thankful to the District Educational Officers, Mandal Educational Officers and head teachers for making this mission possible. I also thank the institutions and organizations which have given their time in the development of this text book. I am grateful to the office of the Commissioner and Director of School Education, (A.P) and Vidya Bhawan Society, Udaipur, Rajasthan for extending co-operation in developing this text book. In the endeavor to continuously improve the quality of our work, we welcome your comments and suggestions in this regard.

Place : Hyderabad
Date : 03 December 2012

Director
SCERT, A.P., Hyderabad

(0)

9th Class

Foreword

Education is a process of human enlightenment and empowerment. Recognizing the enormous potential of education, all progressive societies have committed themselves to the Universalization of Elementary Education with an explicit aim of providing quality education to all. As the next step, universalization of Secondary Education has gained momentum.

The secondary stage marks the beginning of the transition from functional mathematics studied upto the upper primary stage to the study of mathematics as a discipline. The logical proofs of propositions, theorems etc. are introduced at this stage. Apart from being a specific subject, it is to be treated as a concomitant to any subject involving analysis as reasoning.

I am confident that the children in our state of Andhra Pradesh learn to enjoy mathematics, make mathematics a part of their life experience, pose and solve meaningful problems, understand the basic structure of mathematics by reading this text book.

For teachers, to understand and absorb critical issues on curricular and pedagogic perspectives daily focusing on learning in place of marks, is the need of the hour. Also coping with a mixed class room environment is essentially required for effective transaction of curriculum in teaching learning process. Nurturing class room culture to inculcate positive interest among children with difference in opinions and presumptions of life style, to infuse life into knowledge is a thrust in the teaching job.

The afore said vision of mathematics teaching presented in Andhra Pradesh State Curriculum Framework work (APSCF - 2011) has been elaborated in its mathematics position paper which clearly lays down the academic standards of mathematics teaching in the state. The text books make an attempt to concretize all the sentiments.

The State Council for Education Research and Training Andhra Pradesh appreciates the hard work of the text book development committee and several teachers from all over the state who have contributed to the development of this text book. I am thankful to the District Educational Officers, Mandal Educational Officers and head teachers for making this mission possible. I also thank the institutions and organizations which have given their time in the development of this text book. I am grateful to the office of the Commissioner and Director of School Education (A.P) and Vidya Bhawan Society, Udaipur, Rajasthan for extending co-operation in developing this text book. In the endeavor to continuously improve the quality of our work, we welcome your comments and suggestions in this regard.

Place : Hyderabad
Date : 03 December 2012

Director
SCERT, A.P., Hyderabad

(0) Free Distribution by A.P. Government

c) Preface

8th Class

Preface

The Government of Andhra Pradesh has decided to revise the curriculum of all the subjects based on Andhra Pradesh State Curriculum Framework (APSCF - 2011) which recommends that children's life at schools must be linked to their life outside the school. Right to Education (RTE - 2009) perceives that every child who enters the school should acquire the necessary skills prescribed at each level upto the age of 14 years. The introduction of syllabus based on National Curriculum Framework (NCF - 2005) is every much necessary especially in Mathematics and Sciences at secondary level with a national perspective to prepare our students with a strong base of Mathematics and Science.

The strength of a nation lies in its commitment and capacity to prepare its people to meet the needs, aspirations and requirements of a progressive technological society.

The syllabus in Mathematics for these stages i.e. primary, upper primary and secondary is based on structural and spiral approaches. The teachers of secondary school Mathematics have to study the syllabus of classes 8 to 10 with this background to widen and deepen the understanding and application of concepts learnt by pupils in primary and upper primary stages.

The syllabus is based on the structural approach, laying emphasis on the discovery and understanding of basic mathematical concepts and generalizations. The approach is to encourage the pupils to participate, discuss and take an active part in the classroom processes.

The present text book has been written on the basis of curriculum and Academic standards emerged after a thorough review of the curriculum prepared by the APSCERT.

The syllabus has been divided broadly into six areas namely, (1) Number System, (2) Algebra, (3) Arithmetic, (4) Geometry, (5) Measurement and (6) Data Handling. Teaching of the topics related to these areas will develop the skills prescribed in academic standards such as problem solving, logical thinking, mathematical communication, representing data in various forms, using mathematics as one of the disciplines of study and also in daily life situations.

The text book attempts to enhance the endeavor by giving higher priority and space to opportunities for contemplations. There is a scope for discussion in small groups and activities required for hands on experience in the form of 'Do this' and 'Try this'. Teacher's support is needed in setting the situations in the classroom.

Some special features of this text book are as follows

- The chapters are arranged in a different way so that the children can pay interest to all curricular areas in each term in the course of study.

- Teaching of geometry in upper primary classes was purely an intuition and to discover properties through measurements and paper foldings. Now, we have stepped into an axiomatic approach. Several attempts are made through illustrations to understand, defined, undefined terms and axioms and to find new relations called theorems as a logical consequence of the accepted axioms.

Care has been taken to see that every theorem is provided initially with an activity for easy understanding of the proof of those theorems.

- Continuous Comprehension Evaluation Process has been covered under the tags of 'Try this' and 'Think, Discuss and Write'. Exercises are given at the end of each item of the chapter so that the teacher can assess the performance of the pupils throughout the chapter.

- Entire syllabus is divided into 15 chapters, so that a child can go through the content well in bit wise to consolidate the logic and enjoy the learning of mathematics.

- Colorful pictures, diagrams, readable font size will certainly help the children to adopt the contents and ease this book as theirs.

Chapter (1) Rational numbers under the area of number system deal with how a rational number is different from a fraction. Properties of rational numbers are discussed through illustrative examples. Children have been given an opportunity to see the rational number on a number line, the representation of rational numbers on a number line in decimals and vice versa. In chapter (6) Squares and Square roots, we try to make the child, to understand the perfect squares, properties of square numbers and finding square root of a number by factorisation and long division methods. Cubes and Cube roots are also discussed with various illustrative examples.

Chapters (2) (4) (11) and (12) deal with Algebra. In the chapter Linear Equation in one variable, the child is given an opportunity to identify a variable in a verbal problem and finding its value through transposition method. In the chapter Exponents and Powers, some algorithms were given to write bigger numbers in exponential notation. The laws of exponents were discussed with a variety of illustrative examples. In the chapters Algebraic Expression and Factorisation we mostly deal with algebraic expression monomials and binomials. Algebraic identities such as $(a+b)^2 = a^2 + 2ab + b^2$, $(a+b)(a-b) = a^2 - b^2$ and $(x+a)(x+b) = x^2 + (a+b)x + ab$ with geometrical verification are discussed with various values. Factorisation of algebraic expression of these forms are given, along with number of problems to make child to practice.

Chapter (5) Comparing Quantities discussed about ratio, proportion, compound ratio, percentage discount, profit and loss, sales tax/VAT simple interest and compound interest compounded annually, half yearly and quarterly and also application of compound interest formula. Chapter (10) Direct and Inverse Proportion deals with direct proportion, inverse proportion and mixed proportion problems with a variety of daily life situations.

Chapter (15) Playing with Numbers, provides an opportunity to the children to develop algorithms and to find a rule through some patterns of numbers. The divisibility rules are discussed to explore new methods. Ample number of examples and puzzles are given to create interest.

Geometry is discussed with an aim to appreciate the figures the child has seen around him through visualisation and drawing and construction. In the Chapter (3) Constructions of Quadrilaterals, the focus is given for the construction of a unique quadrilateral by revisiting its properties. All models of constructions were given with illustrative examples. In Chapter (8) Exploring Geometrical Figures and Chapter (13) Visualising 3D in 2D, the child has been given enough opportunities to explore various plane figures through 3D.

Data Handling is a key area in which the child will be able to perceive the knowledge of his surroundings through tables, diagrams and graphs. Chapter (7) Frequency Tables and Graphs deals with how to classify the data using tables and to present the data in frequency graphs such as histograms, polygons and O'give curves. Some examples are also given to revise mean, median and mode of an ungrouped data. Alternative methods of finding the values of central tendency and complex problems are discussed.

Finally in chapter (9), the Surface Areas of Plane Figures, we have discussed about finding the area of Trapezium, Quadrilateral, Circle, Circular ring and Sector and also the surface area and volume of cubes and cuboid in Chapter (14).

More the production of good text books does not ensure the quality of education, unless the teachers transform the curriculum the way it is discussed in the text book. The involvement and participation of learner in doing the activities and problems with an understanding is ensured.

Therefore it is expected that the teachers will bring a paradigm shift in the classroom process from mere solving the problems in the exercises routinely to the conceptual understanding, solving of problems with ingenuity.

- Text Book Development Committee

Teachers Hand Book

9th Class

Preface

The Government of Andhra Pradesh has decided to revise the curriculum of all the subjects based on Andhra Pradesh State Curriculum Framework (APSCF - 2011) which recommends that children's life at school must be linked to their life outside the school. Right to Education (RTE - 2009) perceives that every child who enters the school should acquire the necessary skills prescribed at each level up to the age of 14 years. The introduction of syllabus based on National Curriculum Framework - 2005 is every much necessary especially in Mathematics and Science at secondary level with a national perspective to prepare our students with a strong base of Mathematics and Science.

The strength of a nation lies in its commitment and capacity to prepare its people to meet the needs, aspirations and requirements of a progressive technological society.

The syllabus in Mathematics for three stages i.e. primary, upper primary and secondary is based on structural and spiral approaches. The teachers of secondary school Mathematics have to study the syllabus of classes 10 to 11 with this background to widen and deepen the understanding and application of concepts learnt by pupils in primary and upper primary stages.

The syllabus is based on the structural approach, laying emphasis on the discovery and understanding of basic mathematical concepts and generalisations. The approach is to encourage the pupils to participate, discuss and make an active part in the classroom processes.

The present text book has been written on the basis of curriculum and Academic standards envisaged after a thorough review of the curriculum prepared by the APSCERT.

- The syllabus has been divided broadly into six areas namely, Number System, Algebra, Geometry, Measurement, Statistics and Coordinate Geometry. Teaching of the topics related to these areas will develop the skills prescribed in academic standards such as problem solving, logical thinking, mathematical communication, representing data in various forms, using mathematics in one of the disciplines of study and also in daily life situations.

The text book attempts to enhance this endeavor by giving higher priority and space to opportunities for setting. There is a scope for discussion in small groups and activities required for hands-on experience in the form of 'Do this' and 'Try this'. Teacher's report is needed in setting the situations in the classroom.

Some special features of this text book are as follows

- The chapters are arranged in a different way so that the children can pay interest to all curricular areas in each term in the course of study.

- Teaching of geometry in upper primary classes was purely an intuition and to discover properties through measurements and paper foldings. Now, we have stepped into an axiomatic approach. Several attempts are made through illustrations to understand, define, understand terms and axioms and to find new relations called theorems as a logical consequence of the accepted axioms.
- Care has been taken to see that every theorem is provided initially with an activity for easy understanding of the proof of those theorems.
- Continuous Comprehension Evaluation Process has been covered under the tags of 'Try this' and 'Think, Discuss and Write'. Exercises are given at the end of each sub-item of the chapter so that the teacher can assess the performance of the pupils throughout the chapter.
- Entire syllabus is divided into 15 chapters, so that a child can go through the content well in bit wise to consolidate the topic and enjoy the learning of mathematics.
- Some interesting and historical highlights are given under titles of Brain teasers, Do you know! will certainly help the children for creative thinking.
- Colourful pictures, diagrams, readable font size will certainly help the children to adopt the contents and care this book as their.

Chapter (1) Real Numbers under the area number system and irrational numbers in detail. The child can visualise the rational and irrational numbers by the representation of them on number line. Some history of numbers is also added e.g. value of π to create interest among students. The representation of real numbers on the number line through successive magnification help to visualise the position of a real number with a non-terminating recurring decimal expansion.

Chapter (2) Polynomials and Factorisation under the algebra area deal with polynomials in one variable and discussed about how a polynomial is different from an algebraic expression. Factorisation of polynomials using remainder theorem and factor theorem is widely discussed with more number of illustrations. Factorisation of polynomials were discussed by splitting the middle term with a reason behind it. We have also discussed the factorisation of some special polynomials using the identities. Will help the children to counter various types of factorisation. Chapter (3) Linear equations in two variables under the same area will enable the pupil to discover through illustrative examples the unifying face of mathematical structure which is the ultimate objective of teaching mathematics as a system. This chapter links the ability of finding unknown with every day experience.

There are 7 chapters of Geometry i.e. (3, 4, 7, 8, 11, 12, and 13) were kept in this book. All these chapters emphasise learning geometry using reasoning, intuitive understanding and insightful personal experience of meanings. It helps in communicating and solving problems and obtaining

new relations among various plane figures. Development geometry historically through centuries is given and discussed about Euclid's contribution in development of plane geometry through his collection "The Elements". The activities and theorems were given on angles, triangles, quadrilaterals, circles and areas. It will develop induction, deduction, analytical thinking and logical reasoning. Geometrical constructions were presented in such a way that the use of an ungraduated ruler and a compasses are necessary for a perfect construction of geometrical figures.

Chapter (5) deals with coordinate geometry as an alternate approach to Euclidean geometry by means of a coordinate system and associated algebra. Emphasis was given to plot ordered pairs on a cartesian plane (Graph) with a wide variety of illustrative examples. Chapter (9) statistics deals with importance of statistics, collection of statistical data i.e. grouped and ungrouped, illustrative examples for finding mean, median and mode of a given data was discussed by taking daily life situation.

Chapter (14) Probability is entirely a new chapter for secondary school students was introduced with wide variety of examples which deal with finding probable chances of success in different fields, and mixed proportion problems with a variety of daily life situations. Chapter (10) surface areas and volumes we discussed about finding curved (lateral) surface area, total surface area and volume of cylinder, cone and sphere. It is also discussed the relation among these solids in finding volumes and derive their formulae.

Chapter (15) Proofs in mathematics will help the students to understand what is a mathematical statement and how to prove a mathematical statement in various situations. We have also discussed about axiom, postulate, conjecture and the various stages in proving a theorem with illustrative examples.

The success of any course depends not so much on the syllabus as on the teacher and the teaching methods he employs. It is expected that all concerned with the improving of mathematics education would extend their full cooperation in this endeavor.

More the production of good text books does not ensure the quality of education, unless the teachers transact the curriculum the way it is discussed in the text book. The involvement and participation of learner in doing the activities and problems with an understanding is ensured. Therefore it is expected that the teachers will bring a paradigm shift in the classroom process from mere solving the problems in the exercises routinely to the conceptual understanding, solving of problems with ingenuity.

Text Book development committee

d) Unit division

8th Class

Mathematics VIII Class

Contents

Chapter No.	Contents	Syllabus to be Covered during	Page No.
1	Rational Numbers	June	1-33
2	Linear Equations in One Variable	July	34-58
3	Construction of Quadrilaterals	July, August	59-80
4	Exponents and Powers	August	81-95
5	Comparing Quantities using Proportion	August	96-121
6	Square Roots and Cube Roots	September	122-147
7	Frequency Distribution Tables and Graphs	September/ October	148-180
8	Exploring Geometrical Figures	October	181-198
9	Area of Plane Figures	November	199-230
10	Direct and Inverse Proportions	November/ December	231-247
11	Algebraic Expressions	December	248-266
12	Factorisation	January	267-281
13	Visualizing 3-D in 2-D	February	282-296
14	Surface Areas and Volumes (Cube-Cuboid)	February/ March	297-310
15	Playing with Numbers	March	311-336

Free Distribution by A.P. Government

(ix)

9th Class

CONTENTS

Chapter No.	Contents	Syllabus to be Covered during	Page No.
1	Real Numbers	June	1-26
2	Polynomials and Factorisation	July	27-58
3	The Elements of Geometry	July	59-70
4	Lines and Angles	July, August	71-106
5	Co-Ordinate Geometry	August	107-123
6	Linear Equation in Two variables	September	124-147
7	Triangles	September/ October	148-173
8	Quadrilaterals	October	174-193
9	Statistics	November	194-213
10	Surface areas and Volumes	November/ December	214-243
11	Areas	December	244-259
12	Circles	January	260-279
13	Geometrical Constructions	February	280-291
14	Probability	February/ March	292-309
15	Proofs in Mathematics	March	310-327

(ix)

Free Distribution by A.P. Government

e) Syllabus

8th Class

104		105		106			
SYLLABUS		Mathematics VIII		Mathematics VIII			
Number System: (50 hrs) (i) Playing with numbers • Writing and understanding a 2 and 3 digit number in generalised form (100 × 10 + x where x, a, b can be only digits (0-9) and engaging with various puzzles concerning this. (Like finding the missing numbers represented by alphabets in problems involving any of the four operations) • Number puzzles and games • Understanding the logic behind the divisibility tests of 2, 3, 4, 5, 6, 7, 8, 9 and 11 for a two or three digit number expressed in the general form. (ii) Rational Numbers • Properties of rational numbers, including identities • Using general form of expression to describe properties, Approximation of properties. • Representation of rational numbers on the number line • Between any two rational numbers there lies another rational number (Making children see that if we take two rational numbers, then unlike for whole numbers, in this case you can keep finding more and more numbers that lie between them.) • Representation of rational numbers as decimal and vice versa (demonstration after 10, 100, ...) • Consolidation of operations on rational numbers. • Word problems on rational numbers (all operations) • Word problems (higher logic, of operations, including like and unlike area) (iii) Square numbers, cube numbers, Square roots, Cubes, Cube roots. • Square numbers and square roots • Square roots using factor method and division method for numbers containing no more than 4 digits and no more than 2 decimal places		Algebra (20 hrs) (i) Exponents & Powers (ii) Algebraic Expressions (iii) Linear Equations in one variable (iv) Factorisation (v) Factorisation • Factorisation (simple cases only) • Factorisation by taking out common factor. • Factorisation by grouping the terms. • Factorisation by using identities. • Factors of the form (x + a)(x + b) • Division of algebraic expression		Arithmetic (20 hrs) (i) Comparing Quantities using proportion (ii) Direct and Inverse proportion (iii) Geometry (40 hrs) (i) Construction of Quadrilaterals (ii) Representing 3-D in 2-D (iii) Exploring Geometrical Figures (iv) Comparing Quantities using proportion • Compound ratio - Word problems. • Problems involving applications on percentages, profit & loss, overhead expenses, Discount, tax. (Multiple transactions) • Difference between simple and compound interest (compounded yearly up to 3 years or half yearly up to 3 steps only). Applying the formula for compound interest through patterns and using it for simple problems. (v) Direct and Inverse proportion • Direct variation - Simple and direct word problems. Inverse variation - Simple and direct word problems. Mixed problems on direct, inverse variation • Time & work problems- Simple and direct word problems • Time & distance- Simple and direct word problems (vi) Construction of Quadrilaterals • Review of quadrilaterals and their properties. • Construction of quadrilaterals, given with • Four sides and one angle • Four sides and one diagonal • Two adjacent sides, three angles • Three sides and two diagonals. • Three sides and two angles in between them are given • Construction of special types of quadrilaterals with two diagonals. (vii) Representing 3-D in 2-D • Identify and Match pictures with objects (more complicated e.g. nested, joint 2-D and 3-D shapes (not more than 2). • Drawing 2-D representation of 3-D objects (Continued and extended with isometric sketches). • Counting vertices, edges & faces & verifying Euler's relation for 3-D figures with flat faces (cubes, cuboids, triangular prism and pyramids)		Statistics (15 hrs) (i) Area of Plane Figures (ii) Surface areas and Volumes (iii) Surface areas and Volumes (iv) Area of Plane Figures • Area of a triangle using Heron's formula (without proof) and its application in finding the area of a quadrilateral. • Area of a trapezium • Area of the quadrilateral and other polygons. • Area of the circle & circular parts. (v) Surface areas and Volumes • Surface area of a cube, cuboid • Concept of volume, measurement of volume using cubic units, volume of a cube, cuboid • Volume and capacity (vi) Exploring Geometrical Figures • Congruent figures • Similar figures • Symmetry in geometrical figures w.r.t. to triangles, quadrilaterals and circles. (vii) Frequency Distribution Tables and Graphs • Revision of Mean, Median and Mode of ungrouped data. • Determination of mean by deviation method. • Scope and necessity of grouped data. • Preparation of frequency distribution tables • Cumulative frequency distribution tables • Frequency graphs (histogram, frequency polygon, frequency curve, cumulative frequency curve)	
Free Distribution by A.P. Government		Free Distribution by A.P. Government		Free Distribution by A.P. Government			

9th Class

343

SYLLABUS

Number System (50 hrs) (i) Real numbers	<ul style="list-style-type: none"> Review of representation of natural numbers, integers, and rational numbers on the number line. Representation of terminating / non-terminating recurring decimals, on the number line through successive magnification. Rational numbers as recurring / terminating decimals. Finding the square root of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ correct to 6-decimal places by division method Examples of non-recurring / non-terminating decimals such as 1.010101010111— 1.121121121112— and $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}$, $\sqrt{3}$ and their representation on the number line. Existence of each real number on a number line by using Pythagorean result. Concept of a Surd. Rationalisation of surds.
Algebra (20 hrs) (i) Polynomials (ii) Linear Equations in Two Variables	<ul style="list-style-type: none"> Definition of a polynomial in one variable, its coefficients, with examples and counter examples, its terms, zero polynomial. Constant, linear, quadratic, cubic polynomials; monomials, binomials, trinomials. Zero roots of a polynomial / equation. State and motivate the Remainder Theorem with examples and analogy to positive integers (motivate). Statement and verification of the Factor Theorem. Factorization of $ax^2 + bx + c$ where a, b, c are real numbers and of cubic polynomials using the Factor Theorem.

Free Distribution by A.P. Government

346 IX-CLASS MATHEMATICS

SYLLABUS

Number System (50 hrs) (i) Real numbers	<ul style="list-style-type: none"> Review of representation of natural numbers, integers, and rational numbers on the number line. Representation of terminating / non-terminating recurring decimals, on the number line through successive magnification. Rational numbers as recurring / terminating decimals. Finding the square root of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ correct to 6-decimal places by division method Examples of non-recurring / non-terminating decimals such as 1.010101010111— 1.121121121112— and $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ etc. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}$, $\sqrt{3}$ and their representation on the number line. Existence of each real number on a number line by using Pythagorean result. Concept of a Surd. Rationalisation of surds.
Algebra (20 hrs) (i) Polynomials (ii) Linear Equations in Two Variables	<ul style="list-style-type: none"> Definition of a polynomial in one variable, its coefficients, with examples and counter examples, its terms, zero polynomial. Constant, linear, quadratic, cubic polynomials; monomials, binomials, trinomials. Zero roots of a polynomial / equation. State and motivate the Remainder Theorem with examples and analogy to positive integers (motivate). Statement and verification of the Factor Theorem. Factorization of $ax^2 + bx + c$ where a, b, c are real numbers and of cubic polynomials using the Factor Theorem.

Free Distribution by A.P. Government

347

SYLLABUS

(i) Lines and Angles	<ul style="list-style-type: none"> (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse. (Prove) If two lines intersect, the vertically opposite angles are equal. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines. (Motivate) Lines, which are parallel to given line, are parallel. (Prove) The sum of the angles of a triangle is 180°. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.
(ii) Triangles	<ul style="list-style-type: none"> (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence). (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence). (Motivate) Two triangles are congruent if three sides of one triangle are equal to three sides of the other triangle (SSS Congruence). (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal respectively to the hypotenuse and a side of the other triangle. (Prove) The angles opposite to equal sides of a triangle are equal. (Motivate) The sides opposite to equal angles of a triangle are equal. (Motivate) Triangle inequalities and relation between 'angle and facing side' inequalities in a triangle.

Free Distribution by A.P. Government

348 IX-CLASS MATHEMATICS

(iv) Quadrilaterals	<ul style="list-style-type: none"> (Prove) The diagonal divides a parallelogram into two congruent triangles. (Motivate) In a parallelogram opposite sides are equal and conversely. (Motivate) In a parallelogram opposite angles are equal and conversely. (Motivate) A quadrilateral is a parallelogram if one pair of its opposite sides are parallel and equal. (Motivate) In a parallelogram, the diagonals bisect each other and conversely. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and (motivate) its converse.
(v) Area	<ul style="list-style-type: none"> Review concept of area, area of planar regions. Recall area of a rectangle. Figures on the same base and between the same parallels. (Prove) Parallelograms on the same base and between the same parallels have the same area. (Motivate) Triangles on the same base and between the same parallels are equal in area and its converse.
(vi) Circles	<ul style="list-style-type: none"> Through examples, arrive at definitions of circle related concepts radius, circumference, diameter, chord, arc, subtended angle. (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse. (Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and conversely, the line drawn through the centre of circle to bisect a chord is perpendicular to the chord.

Free Distribution by A.P. Government

(vii) Constructions	<ul style="list-style-type: none"> Construction of a triangle given its base, sum / difference of the other two sides and one base angles. Construction of a triangle when its perimeter and base angles are given. Construct a circle segment containing given chord and given an angle.
Mensuration (15 hrs) (i) Surface Areas and Volumes	<ul style="list-style-type: none"> Surface Areas and Volumes Revision of surface area and volume of cube, cuboid. Surface area of cylinder, cone, sphere, hemisphere. Volume of cylinder, cone, sphere (including hemisphere) and right circular cylinder/cones.
Statistics and Probability (15 hrs) (i) Statistics (ii) Probability	<ul style="list-style-type: none"> Statistics Revision of ungrouped and grouped frequency distributions. Mean, Median and Mode of ungrouped frequency distributions (weighted scores). Probability Field of probability using data through experiments. Notion of chance in events like tossing coins, dice etc. Tabulating and counting occurrences of 'through 6' in a number of throws.

Free Distribution by A.P. Government

Proofs in Mathematics (5 hrs) (i) Proofs in Mathematics	<ul style="list-style-type: none"> Comparing the observation with that for a coin. Observing strings of throws, notion of randomness. Consolidating and generalizing the notion of chance in event like tossing coins, dice etc. Visual representation of frequency outcomes of repeated throws of the same kind of coins or dice. Throwing a large number of identical dice/coins together and aggregating the result of the throws to get large number of individual events. Observing the aggregating numbers over a large number of repeated events. Comparing with the data for a coin. Observing strings of throws, notion of randomness.
---	--

Free Distribution by A.P. Government

f) Academic Standards

8th Class

357

Academic Standards

Academic standards are clear statements about what students must know and be able to do. The following are categories on the basis of which we lay down academic standards

Problem Solving

Using concepts and procedures to solve mathematical problems

(a) Kinds of problems:

Problems can take various forms- puzzles, word problems, pictorial problems, procedural problems, reading data, tables, graphs etc.

(b) Problem Solving

- Reads problems
- Identifies all pieces of information/data
- Separates relevant pieces of information
- Understanding what concept is involved
- Recalling of (synthesis of) concerned procedures, formulae etc.
- Selection of procedure
- Solving the problem
- Verification of answers of raiders, problem based theorems.

(c) Complexity:

The complexity of a problem is dependent on

- Making connections (as defined in the connections section)
- Number of steps
- Number of operations
- Context unraveling
- Nature of procedures

Reasoning Proof

- Reasoning between various steps (involved invariably conjecture).
- Understanding and making mathematical generalizations and conjectures

Free Distribution by A.P. Government

358

Mathematics VIII

- Understands and justifies procedures- Examining logical arguments.
- Understanding the notion of proof
- Uses inductive and deductive logic
- Testing mathematical conjectures

Communication

- Writing and reading, expressing mathematical notations (verbal and symbolic forms)
- Ex: $3 + 4 = 7$, $3 < 5$, $n_1 + n_2 = n_3 + n_4$, Sum of angles in a triangle = 180°
- Creating mathematical expressions
- Explaining mathematical ideas in her own words like- a square is closed figure having four equal sides and all equal angles
- Explaining mathematical procedures like adding two digit numbers involves first adding the digits in the units place and then adding the digits at the tens place/ keeping in mind carry over.
- Explaining mathematical logic

Connections

- Connecting concepts within a mathematical domain- for example relating adding to multiplication, parts of a whole to a ratio, to division. Patterns and symmetry, measurements and space
- Making connections with daily life
- Connecting mathematics to different subjects
- Connecting concepts of different mathematical domains like data handling and arithmetic or arithmetic and space
- Connecting concepts to multiple procedures

Visualization & Representation

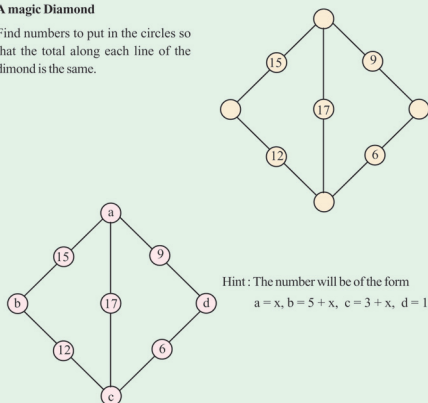
- Interprets and reads data in a table, number line, pictograph, bar graph, 2-D figures, 3-D figures, pictures
- Making tables, number line, pictograph, bar graph, pictures.
- Mathematical symbols and figures.

g) Puzzles

8th Class

A magic Diamond

Find numbers to put in the circles so that the total along each line of the diamond is the same.



where x is any number and the total along each line will be $20 + 2x$
for example if $x = 1$, then $a = 1$, $b = 6$, $c = 4$, $d = 12$ and each line total will be 22.

9th Class

h) Do you know?

Do You Know?

Making an 8×8 Magic Square

Simply place the numbers from 1 to 64 sequentially in the square grids, as illustrated on the left. Sketch in the dashed diagonals as indicated. To obtain the magic square on the bottom, replace any number which lands on a dashed line with its complement (two numbers of a magic square are complements if they total the same value as the sum of the magic's square smallest and largest numbers).

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

64	2	3	61	60	6	7	57
9	55	54	12	13	51	50	16
17	47	46	20	21	43	42	24
40	26	27	37	36	30	31	38
32	34	35	29	28	38	39	25
41	23	22	44	45	19	18	48
49	15	14	52	53	11	10	56
8	58	59	5	4	62	63	1

* A magic square is an array of numbers arranged in a square shape in which any row, column total the same amount. You can try more such magic squares.

i) Brainteaser

9th Class

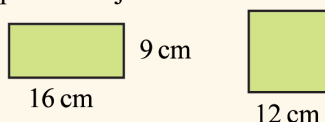
Brain teaser

1. Creating triangles puzzle



Add two straight lines to the above diagram and produce 10 triangles.

2. Take a rectangular sheet of paper whose length is 16 cm and breadth is 9 cm. Cut it in to exactly 2 pieces and join them to make a square.



j) Highlight from History

8th Class

Highlight from History

George Polya (1887 - 1985)

Over the years, many have thought about the question whether the art of problem solving can be taught or is it a talent possessed by only a few? An effective and definite answer was given by the late George Polya. He maintained that the skill of problem solving can be taught.

Polya was born in Hungary in 1887 and received his Ph.D. in mathematics from the University of Budapest. He taught for many years at the Swiss Federal Institute of Technology in Zurich.

Among the numerous books that he wrote he seemed most proud of "How to Solve It" (1945) which has sold nearly one million copies and has been translated into 17 languages.

Polya's Four principles of Problem solving

I. Understand the problem

This principle seems so obvious that it need not be mentioned. However students are often asymmetrical in their efforts to solve a problem because they don't understand it fully or even in part. Teachers should ask students such questions as

- Do you understand all the words used in stating the problem? If not, look them up in the index, in a dictionary or wherever they can be found.
- What are you asked to find or show can you restate the problem in your own words.
- Is there yet another way to state the problem
- What does (they) really mean?
- Could you work out some numerical examples that would help make the problem clear?
- Could you think of a picture or diagram that might help you to understand the problem.
- Is there enough information to enable you to find a solution.
- Is there extraneous information?
- What do you really need to know to find a solution.

II. Devise a plan

Devising a plan for solving a problem once it is fully understood may still require substantial effort. But don't be afraid to make a start you may be on the right track. There are often many reasonable ways to try to solve a problem and the successful idea may emerge only gradually after several unsuccessful trials. A partial list of strategies include:

- guess and check
- look for a pattern
- make an orderly list
- draw a picture
- think of the problem as particularly solved
- think of a similar problem already solved
- eliminate possibilities
- solve simpler problem
- solve an equivalent problem
- solve an analogous problem
- use symmetry
- use a model
- consider special cases
- work backward
- use direct reasoning
- use a formula
- solve an equation
- be ingenious

III. Carryout the plan

Carrying out the plan is usually easier than devising the plan. In general you need to be careful and persistent, given that you have the necessary skills. If a plan does not work immediately be persistent. If it still doesn't work, discard it and try a new strategy. Don't be misled this is the way mathematics is done, even by professionals.

IV. Look back

Much can be gained by looking back a completed solution to analyze your thinking and ascertain just what was the key to solving the problem. This is how we gain "Mathematical power", the ability to come up with good ideas for solving problems never encountered before.



George Polya (1887-1985)

9th Class

Highlight from History

"The Wonder of Discovery is especially keen in childhood"

How a child become Ramanujan a great mathematician at all the time?

Ramanujan

Something new. As a boy he impressed his classmates, senior students and teachers with his insight and intuition.

One day in an arithmetic class on division the teacher said that if three bananas were given to three boys, each boy would get a banana and he generalized this idea. Then Ramanujan asked "If no banana is distributed to no student will every one still get a banana?"

Ramanujan's math ability won several friends to him. Once his senior student posed a problem "If $\sqrt{x} + y = 7$ and $x + \sqrt{y} = 11$, what are x and y ?"

Immediately Ramanujan replied $x = 9$ and $y = 4$. His senior was impressed and became a good friend to him.

In his school days, along with school homeworks, and so on.

Ramanujan worked with some patterns out of his interest.

$$3 = \sqrt{9} = \sqrt{1+8}$$

$$= \sqrt{1+(2+4)}$$

$$= \sqrt{1+2+4}$$

$$= \sqrt{1+2+4+9}$$

$$= \sqrt{1+2+4+(3+5)}$$

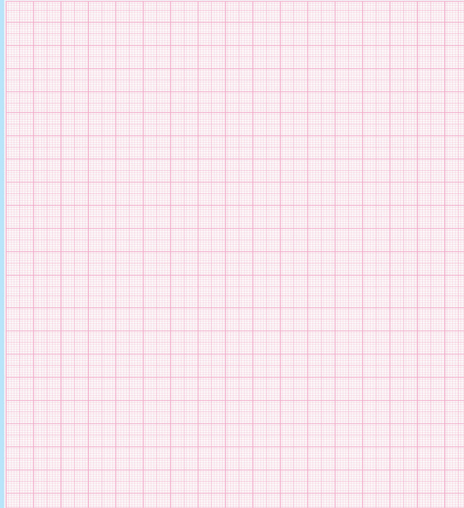
$$\text{and so on...}$$

Srinivasa Aiyangar Ramanujan is undoubtedly the most celebrated Indian Mathematical genius. He was born in a poor family at Erode in Tamil Nadu on December 22, 1887. Largely self taught, he studied on "Lambert's Trigonometry" at the age of 13, and at the age of 15, his senior friends gave him synopsis of Elementary results in pure and Applied mathematics by George Carr. He used to write his ideas and results on loose sheets. His filled note books are now famous as "Ramanujan's Frayed note books". Though he had no qualifying degree, the university of Madras granted him a monthly Scholarship of Rs. 75 in 1913. He had sent papers of 120 theorems and formulae to great mathematician G.H. Hardy (Cambridge University, London). They have recognized these as a worth piece and invited him to England. He had worked with Hardy and others and presented numerical theories on numbers, which include circle method in number theory, algebra inequalities, elliptical functions etc. He was second Indian to be elected fellow of the Royal Society in 1918. He became first Indian elected fellow of Trinity college, Cambridge. During his illness he also never forgot to think about numbers. He remarked the total number of Hardy, 1729 is a singularly unexceptional number. Is the smallest integer that can be represented in two ways by the sum of two cubes, $1729 = 1^3 + 12^3 = 9^3 + 10^3$. Unfortunately, due to tuberculosis he died in Madras on April 26, 1920. Government of India recognised him and released a postal stamp and declared 2012 as "Year of Mathematics" on the eve of his 125th birth anniversary.

k) Wonderful Circle (Back Page)

9th Class

Graph Paper



Wonderful Circle

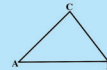
Constructing the Nine-Point Circle of a triangle

The circle which passes through the feet of the perpendiculars, dropped from the vertices of any triangle on the sides opposite them, passes also through the midpoints of these sides as well as through the midpoints of the segments which join the vertices to the point of intersection of the perpendiculars. Do you know all that? This circle is called the Nine-Point Circle. This Nine-Point circle result was known to Leonard Euler 1765, but was rediscovered by German Mathematician Karl Feuerbach in 1822.

Constructing the nine-point circle is a good test of your construction skills and your ability.

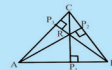
Just follow the instructions and try this

Step 1



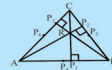
Construct a large scalene triangle on a sheet of white paper. Label it $\triangle ABC$.

Step 2



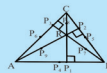
Construct the altitude to each side of the triangle and label the points of intersection with the sides P_1 , P_2 , and P_3 . Name the orthocenter as R .

Step 3



Construct the midpoint of each side of the triangle. Label the points P_4 , P_5 , and P_6 , so that P_4 is the midpoint of AB and P_5 is the midpoint of BC .

Step 4



Construct the midpoints of BR , CR and AR and label the points P_7 , P_8 and P_9 so that P_7 is the midpoint of BR and P_8 is the midpoint of CR .

Step 5



Construct the line segments connecting points P_1 to P_8 , P_2 to P_7 and P_3 to P_9 . They should all intersect in one point. Mark that point as "O".

Step 6



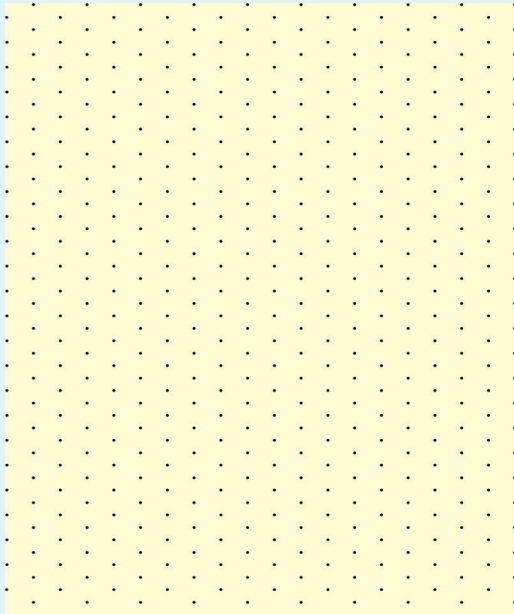
Construct a circle with radius OP_1 and center at point O. It should pass through all nine points: P_1 , P_2 , P_3 , P_4 , P_5 , P_6 , P_7 , P_8 , P_9 .

This is the wonderful circle. You might have observed that how "compass" play a major role in this Geometrical construction.

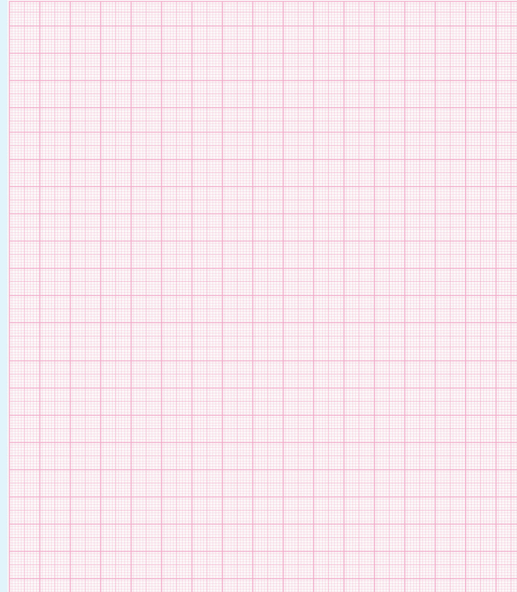
l) Graph Sheet / Isometricsheet

8th Class

Isometric dot paper



Graph Paper



Discuss : How these special features in new text books are helpful to the children of 8th and 9th classes?

♦ **A review of 8th Class New Text Book :**

The new text book has 352 pages where as the used in the past contain 311 pages. The size of the book is increased as the chapters were developed with more illustrations, tables, Graphs etc., The number of chapters remains same as 15. Taken care to reduce the content without changing the contextual part. All the topics were arranged in a systematic and logical order following spiral approach. The chapter include sufficient material for children to enjoy and donot feel boredom in solving problems. Chapters like "Sets`relations", Linear equations and in equations, some position of commercial mathematics were deleted from the syllabus as these are to be learned throughly in higher classes. The rational numbers were discussed in detailed manner with more illustrations and examples to understand their properties. Learning of Geometry in the class was confined to construction of quadrilaterals, Exploring geometrical ideas were stifled to next higher classes. Concepts like Sales Tax, VAT etc were introduced along with ratio and proportion. A chapter 'playing with numbers' was introduced in a different manner in which children will explore fomulae based on pattern and dissibily rules of number with reasoning. In Algebra, more emphasis is given to understand Linear equations, exponents, Algebraic expressions with more classroom activities and analysis. Factorisation and special products were shifted to next class. In every chapter more examples were given to understand the problem solving approach in solving problems. By reducing the member of exercises, the text book writers tried to reduce the stress in learning mathematics. All concepts in a chapter were linked with small exercises under 'Dothis', try these. There exercises will help the teachers and students every stage. The questions were also useful in building up the confidence individually as well as in groups.

♦ **A review of 9th Class New Text Book :**

The new 9th class text book has been changed throughly componing to the previous book. Geometry has been given more importance as per the prescribed syllabus and physical and mental age of the children at this level. This year Geometry includes Elements from Enclidian Geometry, lines and angles, Triangles, quadrilaterals, Areas, Cricles and Geometrical constructions witht justifications. All are were linked with concepts learned previously in 8th class. But the understanding the activities followed by logical conclusions which leads to proving of the theorems were given. Many worked examples were given to understand the application of theorems in solving problems.

The new text book consists of 342 pages comparring to the previous one with 439 pages. Motst of the unxlated topics were removed such us logarithms, squareroots of algebraic expressions, eyclic expressions, quadratic equations, sets, relations, Linear equations and ineqations, matrices and computing. As per the Curriculum and syllabus prescribed at this level, every topic should have relevance to the context. Some new concepts like remainder

theorem, factor theorem, co-ordinate geometry were associated and statistics chapter was much strengthened which includes finding mean, median and mode of semigrouped data and through graphs. Probability was introduced as a new chapter and understanding of mathematical proofs were given with daily life examples. The new text book reduces the burden of much content and includes problems which makes children to think and discuss in peer groups.

The new text books were good in size, shape and colourful. Some additional information about History of Mathematicians, problem solving approaches were included. Also added some puzzles and Brain teasers to create interest among children for joyful learning of Mathematics.

Conclusion :

The text books written on the basis of new curriculum and syllabus will provide ample scope to learn all mathematical concepts in all the three classes 8th, 9th and 10th classes.

The teachers should plan accordingly with the concepts and syllabus with their innovative teaching strategies to provide new ideas to the children which reads mathematician among them.

The National policy on Education, 1986 states that Mathematics should be visualised as the vehicle to train a child to think, reason, analyse and articulate logically. Apart being a specific subject it should be treated as concomitant to any subject involving analysis and reason. The national curriculum framework for school education (NCFSE-2005) echoes such statements, but despite this mathematics education has remained much the same, focussed on narrow aims.

- SCF 2011

Chapter - 3

Analysis of a chapter in the New Text Book

Unit Plan

Arithmetic

8th Class

Unit : Comparing quantites with proportion

16 periods (12 hours)

(Chapter - 5)

S.No.	Sub topic	Concepts	Periods	Material
1.	Ratio (2)	i) Concept of ratio	1	Charts, news papers
		ii) Golden ratio	2	Golden ratio pictures
		iii) Compund ratio		related
		iv) Peoblems on Compound ratio	1	Monuments
2.	Percentage (3)	i) Comparing quantities with percentage	1	Charts
		ii) Finding increase and decrease of quantities with percentage	3	
		iii) Varbal problems related to ratio and percentages	1	
		iv) Estimating percentage	1	
3.	Discount (2)	i) Concept of discount	1,2	Charts
		ii) Verbal problems on discount	1	
4.	Profit and Loss (4)	i) Concepts of profit and loss	1	Charts
		ii) Finding differences between profit and loss	4	
		iii) Calculation of profit and loss percentage	1	
		iv) Verbal problems relating to profit and loss		

S.No.	Sub topic	Concepts	Periods	Material
5.	Sales Tax and VAT (1)	i) Concept of VAT	1	Charts Modle Bills
		ii) Verbal Problems	4	
6.	Compound Interest (4)	i) Concept of compound Interest	1	Charts
		ii) Derivation of the formula of Compound interest		
		iii) Calenlation of compound interest per annum or halfyearly	3	Pomphlets of different banks
		iv) Application and verbal problems on compound interest		Documents downloaded from interest

Topic : Arithmetic

Chapter : Companing quantities with proportion

Class : 8th Class

1. Academic Standards Prescribed :-`

- 1) **Problem solving** :
 - Solve problems related to compound ratio.
 - Solve verbal problems.
- 2) **Giving reasons** :-`
 - Finds difference between ratio and percentage.
 - Estimates percentage.
 - Finds difference between profit and loss.
 - Derives formula to find compound interest.
- 3) **Communication** :
 - Express the ideas about compoundratio in ownwords.
 - Explains different methods to solve problems on percentages, profit and loss and compound interest.
 - Explains the logic behind deriving the formula of compoud interest.
- 4) **Connection** :-`
 - Connects division with ratio and percentage.
 - Connects percentage with profit and loss, discount and VAT.

- Connects compound interest and VAT with daily life problems.
 - Connects problems related with profit, loss and compound interest with algebra.
- 5) Representation** :`
- Explains compound ratio using tables.
 - Explains percentages with tables.
 - Prepare bills using discount and VAT.
 - Understand different tables and information given in the chapter.
- 2. Teaching learning material** :
- Textbooks Graphsheets, Charts, Sketches, Bills collected from shops, Brochers distributed by banks pictures of historical monuments.
- 3. Introduction** :`
- Divide whole class into two groups as A and B and create a situation for comparison of different aspects like height, weight etc.
- 4. Teaching Strategies** :`
- The concept of compound ratio is to be introduced by taking an example from dialy life situation.
 - The child has to find the compound ratio of $a : b$ and $C : d$ with examples.
 - Creating a situation in the classroom to konw how the percentage increases and decrease.
 - Motivate the children to participate in groups and whole class activity to solve verbal problems relating to compound ratio and percentage.
 - Discuss the importance of Discount in the classroom with tables and charts.
 - By observing different types of bills students understand the difference between profit and loss.
 - Some textual problem about profit and loss must be showed.
 - Develop the concept of profit and loss with daily life examples and by showing shopping bills.
 - Develop the concepts of VAT and compound interest with bills and Bantery Brochers.
 - Derive the formula of compound interest with pattern and activity.
 - Solving verbal problems in classroom leads to the application.

Implementation of teaching strategies**Ratio :**

- ♦ Refer page 97 and 98 of text books and make the children to understand the tables of investments.
- ♦ Students will comprehend the compound ratio.
- ♦ Group activity is to be given for "Try these" item in page.99.
- ♦ Ask the children to find more pictures related to golden ratio.
- ♦ Students must be provided an opportunity to understand and appreciate the Golden Ratio.
- ♦ Motivate the children to solve problems in exercise 5.1 on their own.

Percentage :

- ♦ Create a situation to develop the concept of ratio through the example in page.99.
- ♦ Encourage children to think and understand different methods given in page no.101.
- ♦ Ask the students to solve problems 7, 11 in the exercise 5.1.

Discount :

- ♦ Display the charts representing prices of articles and discount given by shopping malls as given in page no.102 to make the children to understand concept of discount.
- ♦ Use the item 'try these' in page no.104 to make the children to discuss about discount used in various daily life situations.
- ♦ Make the children to discuss and solve the problems given in page.105.
- ♦ In the items 'Try these' and 'Think discuss and write' to understand how to estimate the percentages.

Profit and Loss :

- ♦ Make the children to observe the situations written on the chart of page.105.
- ♦ Idea of "Profit", "Loss" must be inculcated by observing the above discussion.
- ♦ Ask the children to solve problems given in p.106 and 107 by discussing with friends.
- ♦ Motivate the children to solve problems 8, 9, 10, 11, 12 of exercise 5.2 on their own.

Discount and VAT :

- ♦ Ask the children to collect bills from shops and customers for observing discount on items.
- ♦ Make the children to understand about VAT by taking them to some shops / Hotels / malls.
- ♦ Motivate children to discuss the example problem given in page 109.
- ♦ Ask the children to solve problems 5, 6, 13, and 14 on their own.

Compound interest :

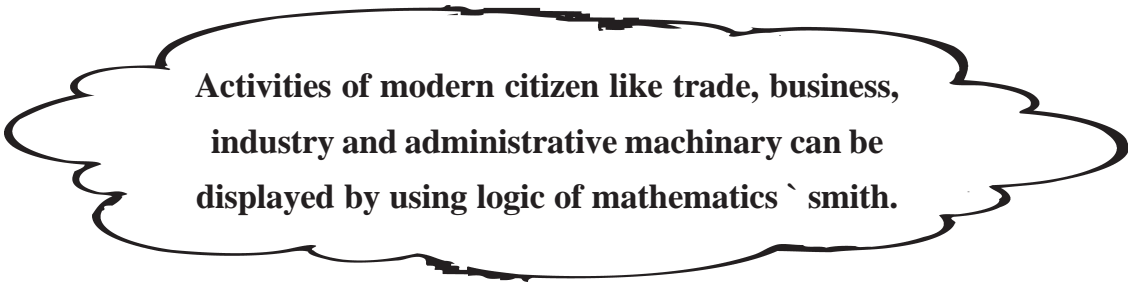
- ♦ Make the children to observe the information given by the Banks on their brochures about deposits, interest rates on loans etc.
- ♦ By discussion on such brochures in the classroom, idea of compound interest must be developed among the students.
- ♦ Generalise the idea of compound interest through examples to arrive the formula to find compound interest (Page.113)
- ♦ Ask the children to solve problems given in Page 114, Page 115
- ♦ Through 15th example on the page no.118, application of compound interest in physical interest in physical science must be highlighted.
- ♦ Make the children to understand the relation between the application of physical science problems with compound interest.
- ♦ Problems 1, 3, 6, 8, 10, 14 and 15 of exercise 5.3 must be solved on the black board, while discussing with the students.

Additional Information :

- ♦ Collecting pamphlets of business organisations and insurance companies and discussing with the children about discount, interest, profit / loss.
- ♦ Inspiring the students to collect beautiful photographs or pictures and to identify the 'Golden Ratio'.

Key Concepts :

Golden Ratio, Compound Ratio, percentage as a ratio, discount, VAT, profit / loss, compound interest, derivation of formula for compound interest.



Activities of modern citizen like trade, business, industry and administrative machinery can be displayed by using logic of mathematics `smith.

Chapter-4

Special Items -An Analysis

1. Verbal problems

Mathematical equation:

Know What + Know how + Know why = Know more

According to a research conducted by Institute of Educational Science, department of Education, U.S.A., the steps to solve verbal problems are as follows:

I. I. Read the Given verbal problem.

- ◆ Understand the problem
- ◆ Understand the mathematical language
- ◆ Identifies the type of the problem
(Construction, proof, graphs, fill the table etc.,)

II. Fill the following chart and solve.

1.	Identify "what are given, What are to be find" in the given verbal problem. <ul style="list-style-type: none"> ◆ Can you understand the given problem and can you explain it in your own words? ◆ Can you explain the given problem in another form? ◆ What is the meaning of important terms in the given problem? ◆ Can you draw a needed figure to solve the problem? 	
2.	To solve the problem what information is needed? <ul style="list-style-type: none"> ◆ Identify whether the given information is helpful to solve the problem ◆ Write the given problem in your own words to identify what to find in the given problem ◆ Write the steps in order ◆ Refer the solutions of identical problems ◆ Write "what do you find" in the given problem in mathematical language 	
3.	Does the given information is sufficient to solve the problem? <ul style="list-style-type: none"> ◆ Whether extra information is given? ◆ Identify the extra information that is needed to solve the problem 	

	<ul style="list-style-type: none"> ◆ Identify which operation, principle or activity to be used to solve the problem ◆ Identify the necessary theorem to solve the problem 	
4.	<p>How is the solution?</p> <ul style="list-style-type: none"> ◆ Prepare a plan/algorithm to solve the problem ◆ Try to find a pattern ◆ Write the steps in order ◆ Draw a relevant figure ◆ Identify what is needed and what is unnecessary. Remove unnecessary things ◆ Using of symmetry ◆ Using of a tool ◆ Identify the relevant activity and implement it ◆ Act accordingly and wisely 	
5.	Solve the Problem	
	Recaptulation:	
6.	What is your answer?	
7.	How did you solve the problem? What was the method?	
8.	Explain the reasons for using that particular method?	

Worksheet / Verbal Problems

1. Resolve into factors: $2y(y + z) - (x + y)(x + z)$

(Note: don't remove the brackets)

2. Find the value of
$$\frac{(1986^2 - 1992)(1986^2 + 3972 - 3)(1987)}{(1983)(1985)(1988)(1989)}$$

3. A right angle triangle with measurement 15cm, 20cm, is rotated throw its hypotenuse. Find the volume and total surface area of so formed bi-conical shape?

4. A cylindrical shaped vessel whose inner radius is 21cm. Is filled with water. A solid sphere whose diameter is 10.5 cm. Is being dipped in the water/vessel. Find the increase in water level?

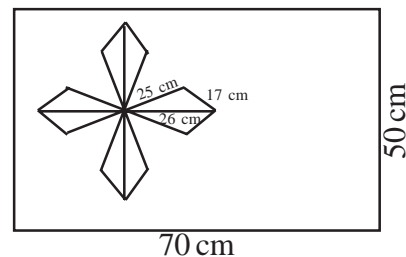
5. a, b, c, x, y, z are real numbers:
$$\frac{a + b + c}{x + y + z}$$

$a^2 + b^2 + c^2 = 25$; $x^2 + y^2 + z^2 = 36$ and $ax + by + cz = 30$, then find the value of $\frac{a^2 + b^2 + c^2}{x^2 + y^2 + z^2}$

6. In a right angle triangle the length of the smallest side is 2003 units. If the lengths of remaining sides are positive integers, find the perimeter of the triangle?

7. Radius of a sphere is 5 cm. Surface area of the sphere is 5 times of the lateral surface area of a cone whose radius is 4 cm. Find the height, volume of the cone ($\pi = \frac{22}{7}$)

8. In the adjacent figure the measurement of each triangle is 26 cm., 17 cm, and 25 cm., figure, as in the Telugu version. All such 8 triangles (forms a design) are drawn in rectangular shaped slate with measurements 50 x 70 cm. Find the area of the design and area of the remaining place?

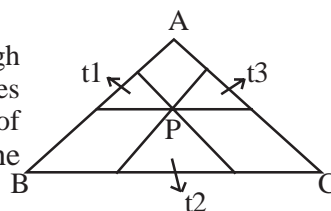


9. Prove that $2222^{5558} + 5555^{2222}$ is divisible by 7?

10. In a rectangle ABCD, AB = 16 Units, BC = 12 units and F & E are points on AB, CD respectively, if AFCE is rhombus, then find the measurement of EF?

11. $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+2013} = ?$

12. A and B are four digit numbers, $a > b$, if a number is reversed, then we will get another number and if $\frac{a+b}{5} = \frac{b-1}{5}$, then find the value of 'b'?
13. $x + y + z + t = 1$
 $x + 3y + 9z + 27t = 81$
 $x + 4y + 16z + 64t = 256$
 Find the value of 'x' if $x + 167y + 167^2z + 167^3t = 167^4$
14.
$$\frac{(1+17)(1+\frac{17}{2})(1+\frac{17}{3}) \dots (1+\frac{17}{9})}{(1+19)(1+\frac{19}{2})(1+\frac{19}{3}) \dots (1+\frac{19}{17})}$$
15. A line parallel to 'x' axis, intersects the graph $y = \frac{x-1}{(x-2)(x-3)}$ at $x=a$, $x=b$ then find the value of $(a-1)(b-1)$?
16. Sides of a right angle triangle are "a and b" and $a > b$. If the angle bisector of right angle divides the given triangle into two similar right angle triangles then find the distance between two Ortho centres of so formed two right angle triangles.
17. Find the remainder when x^{2013} is divided by $(x^2 - 1)$?
18. If $[YE][ME] = [TTT]$ then find the value of $Y+E+M+T$ (YE, ME are two digit numbers and TTT is a three digit number)
19. P is a point inside of triangle ABC. Three lines, which are drawn through the point P and each line is parallel to each side of the triangle, divides the triangle into 6 small parts as shown in the figure. If the areas of smaller triangles $\Delta t_1, \Delta t_2, \Delta t_3$ are 4,9,16 respectively, then find the area of triangle ABC.
20. If the length of sides of right angle triangle are integers then show that radius of the Incircle of the triangle is also an integer.
21. If $x, y \in \mathbb{Z}$ and $x < y, x^2 + y^2 = 2000$ then show that $31 < y < 45$.
22. In triangle ABC, the incircle touches the sides BC, CA and AB at D, E and F respectively. Radius of the incircle is 4 cm. And the lengths of BD, CE, AF are consecutive integers, find the lengths of sides of the triangle.
23. If $(a^2 + b^2)^3 \mid (a^3 + b^3)^2$ and $ab \neq 0$ then find the value of $\frac{a}{b} + \frac{b}{a}$
24. Find the digits in the following addition



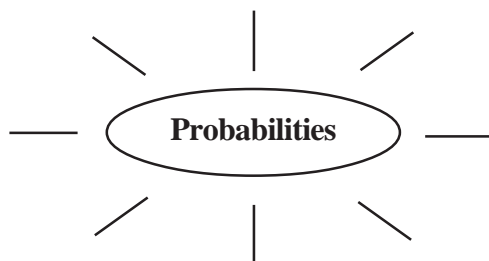
$$\begin{array}{r} \text{T I C K} \\ \text{T O C K} \\ \text{T I C K} \\ \text{T O C K} \\ \hline \text{A} \\ \hline \text{C L O C K} \end{array}$$

(T,I,C,K,O,L,A are non repeated digits)

2. Probability

1. (a) What is Probability ?
- (b) What is the need of learning probability?

What is Probability? (Brain storming)



The world where we live is not stable. All surroundings of human beings are not under his/her control. Some of them are ruled by the nature. For example to reach a certain place, if there are many ways generally we think that which way is safe and in which way we can reach quickly? To perform a cardiac operation to our friend we think that which hospital is better one? How the weather is going to be, for the next 3 days? The train scheduled to start at 10.00 'O' clock will start at right time?

Frequently we will face such questions in our daily life. We show interest in guessing how the things are going to be done in the future. Many reasons will help us to take a decision on what we guess. To take a decision on what we guess, we need the knowledge of Probability. To express the probability we use fractions, decimals or percentages. **Probability is a digital expression to show how the incident will probably occur.**

◆ Uses of Probability :

We take many decisions without our concern by using the probability.

Medical related decisions : To get an operation for our near and dear we will think of about a good hospital , we will bring them to that hospital only and finally we will try to get a most successful doctor in this field.

Sports and Games : To select a player, all his/her records will be taken in to consideration. By making rating on his/her performance we will decide whether he /she is eligible?

Insurance Sector : when the companies extending insurance to a person, his/her age, his/her health grounds, probability of death rate of that particular age group are to be taken into consideration.

Weather forecast : The weather of coming few days will be guessed according to the temperatures of the past few days and cyclones happened in that year.

b) If two dice are rolled two times then the number of possible outcomes = _____

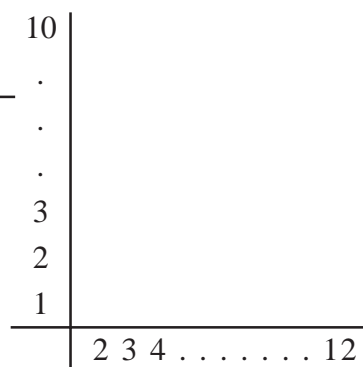
c) If the two dice are rolled three times then the number of possible outcomes = $\frac{108}{(Why?)}$

Roll a dice for 108 times (why is it rolled more times?) then note the sum of the scores on the top faces of dice in the following table:

Possible total	Tally marks	Frequency
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
Total		108

Draw a bar graph by using the above table.

From the bar graph the possible maximum score = _____



Theoretical probability (B) :

Note the possible total score on first two dice in the following table :

2nd dice

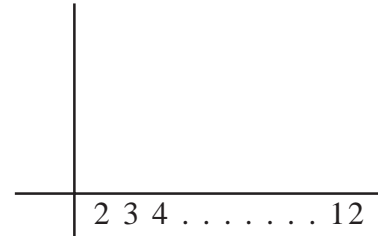
1 st dice	Total	1	2	3	4	5	6
	1						
	2						
	3						
	4						
	5						
	6						

From the table fill the following :

Total	2	3	4	5	6	7	8	9	10	11	12
Number of time occurred											

Draw a bar graph for the above information

The possible maximum score = _____



Discuss the following questions in both the methods (A,B).

1. Do the length of the bars is same? _____
2. Maximum occurred score _____
3. Are all bar graphs symmetrical? _____

Note :

If an experiment is done for many times then the experimental probability closes to the theoretical probability. This concept is the base for statistics and probability. It was established by Bernoulli in 1713.

It is called Law of Large Numbers.

Text Book is important resource for the teacher as well as student. Teacher uses the textbook to know the mathematical concepts that he has to teach, methods of teaching and to prepare lesson plans. Students use the text book to know the concepts and methods.

- SCF 2011

Worksheet / Probability

How the headlines in today's news probably?

Aim/objective : After competition of the activity the participants/trainees can able to express their daily life situations and chance of their occurrence in the language of probability

Groups : Make the participants in to 6-7 groups, each consisting of 4 members

Material : Chart, Sketch pens, Glue, scissors, old news papers

After discussion with the group members, imagine the 5 important news items which will be telecasted at 9.p.m to night (10 minutes)

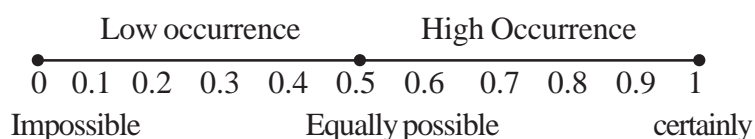
(On board)



After 10 minutes a group come forward and write their first probable news item on the board. This will be discussed in the class and its probable occurrence is noted on the above scale

If it is to note its probable occurrence at more than the above points in the scale the following terms may be used. They are

High occurrence, Low occurrence



Trainees will read the chapter "Probability" (Chapter -4.1), page 292 (10 minutes)

Then the group discuss and rewrite their news items in the order.

Presentation:

Every group will Present their news head lines by using the terms impossible, equally possible, highly possible, certainly, etc.

3. Proofs in Mathematics

1. Un defined terms :

Mathematics is the independent of all other subjects. So to define the terms in this subject, we have to use another mathematical terms. After defining the first term with its help and with the help of other mathematical terms we can now able to define another (one more) term. By using this another term we can define some other termwe have to go in this process only. But In this way when we try to define the terms in mathematics, at some junction we will again reach to the first term. Aristotle proposed that if we wanted to get rid of this situation we would have some undefined terms. Up to 19th century no mathematician had looked into this undefined terms. This undefined terms got important due to Maritz Patch in the year 1882.

Undefined terms are explained with their meaning only but not with definition. We adopted- Point, Line, surface, number, space zero etc., are undefined terms in mathematics. Because of the entry of set theory their number was decreased why because set theory links the different branches of the mathematics.

Definitions:

If we want to define a term first of all we have clear understanding about what it explains. Defining a term means explaining its meaning with other terms.

A good definition consist of (1) the term which is going to be defined is to be mentioned (2) to define it, the undefined or previously defined terms only are to be used (3) the class to which it belongs and how it differs and in which issues it differs from the other terms is to be indicated (4) No scope is to be given for unnecessary issues/terms (5) converse has to be existed.

Ex: A polygon with three sides is a triangle.

Explanation: (1) triangle, the term which is going to be defined, is mentioned (2) it belongs to polygons which is mentioned (3)How it differs with other polygons with respect to sides is also mentioned

Ex: A Polygon with three sides and three angles is a triangle

Explanation: Here three angles is not necessary

2. Axioms :

Any subject which has scientific and logical base is started with universally agreed fundamental concepts. These fundamental concepts don't have proofs. These are to be taken as true and with the help of them some other concept are to be developed. These concepts are agreed by all as true concepts.

In mathematics such concepts without proofs are considered as Axioms. For example (1) part is always bigger than the whole (2) if equal parts are added to equal s then the resulting wholes are also equals.

In geometry (1) through the given two points only one line can be drawn (2) circle can be drawn for any given diameter

But at present we are using all fundamental concepts as axioms.

3. Theorems-Conjectures :

Theorems are statements which are made from deductive method on the basis of Scientific and logical deductions of undefined words, defined words and axioms. All theorems are conditional statements

Conjectures are statements which are made by observing different patterns/configurations in mathematics. All these are conditional statements. These conjectures don't have proofs. They are made by observation only. Whenever a conjecture is proved then it becomes as a theorem.

- Ex:-
- 1) Gold Back conjecture
 - 2) Riemann conjecture
 - 3) 144 is the only square number in Fibonacci numbers

We felt that four colour problem was a conjecture. But it was proved recently

Truly speaking all theorems/conjectures are statements made by observing patterns and by linking the mathematical facts/issues/concepts. Every theorem in the beginning days stated as a conjecture. Proof will be given later. Pythagorean Theorem was also in the beginning stated as a conjecture and later it was proved.

4. Statements :

Statements are sentences which we can decide whether they are true or false but not both. That means every definition is a true statement.

In mathematics all statements are proved, but in other subjects they are just examined or verified only. Mathematical statements are deducted from already known facts. In other subjects they are established from the results and conclusions taken from the experiments or by observing the facts. That is why the theory of relativity is just examined and verified only, but not proved.

5. What is a Proof?

Proof of mathematical statement means the truth value of the statement is being proved with the help of logical and scientific arguments. In this process all the steps which we use are conditional statements. Proof means a universally agreed argument.

Importance of proof:

Let us observe the following examples:

Ex:-Prove that $\frac{1}{1000} - \frac{1}{1001} < \frac{1}{1000000}$

Proof: $\frac{1}{1000} - \frac{1}{1001} = \frac{1001-1000}{1001000} = \frac{1}{1001000}$

But $1001000 > 1000000$ Therefore $\frac{1}{1000} - \frac{1}{1001} < \frac{1}{1000000}$

With the help of calculator or computer it can be checked or verified. If doing so we can escape from establishing the proof. But in mathematics, understanding is important. A machine can answer for a problem but it does not give answer for the question why? Further proof leads to generalisation.

For example we can prove that $\frac{1}{n} - \frac{1}{n+1} < \frac{1}{n^2}$

There is limit for digits in the computers or in calculators, if the limit exceeds they give only approximate values.

6. How is a mathematical statement proved?

There are no special methods to prove mathematical statements. Because of this Fermat's theorem took 300 years to be proved. Still many conjectures are not yet proved. But if we think in the way as shown by Polya we may find some clue.

First we must understand clearly, what is required to prove, and then we should have a rough idea about how to proceed. First we have to understand the statement which has to be proved i.e., we have to know what is our aim? For this purpose you have to question yourself about what is given (Hypothesis) and what is to be proved (Conclusion), what constraints are given. Draw a graph or picture/diagram if it is needed/possible and Hypothesis and conclusion are shown by relevant symbols.

Then the relation between Hypothesis and conclusion is to be identified. If there is no direct relation between these two, we have to try to make a relation with the help of sub objects.

7. How to write a proof?

- i) Draw a diagram which has to be shown all information.
- ii) Write the given part (Hypothesis) with relevant words.
- iii) Write the to be proved part (Conclusion) with relevant words.
- iv) To reach the object, study the diagram for further information/understanding.
- v) Then start to write proof. Explain reason for every step. The following items can be used as reasons. (i) axioms (ii) definitions (iii) given part (iv) statements which are proved earlier.

8. Method of proving :

a) Hypothesis

b) Conclusion

c) Figure

d) Proof steps/statements

Reasons

1) _____

1) _____

2) _____

2) _____

3) _____

3) _____

4) _____

4) _____

5) _____

5) _____

9. Methods of Proofs :

1) Direct Proof : (i) in direct proof, in a theorem $H \rightarrow C$ (Hypothesis-Conclusion) H is taken as true and with logical reasons, by using the known facts we will reach C . If we explain this steps as $H \rightarrow C_1 \rightarrow C_2 \rightarrow C_3 \dots \rightarrow C$, then C_1, C_2, C_3, \dots are used as sub objects/aims.

Ex:- Show that $a^2 < b^2$ if a, b are real numbers and $a < b$.

Proof:

$a < b \Rightarrow a^2 < ab$ (by multiplying with a on both sides)

$\Rightarrow ab < b^2$ (by multiplying with b on both sides)

$\Rightarrow a^2 < ab < b^2$ (associate property)

$\Rightarrow a^2 < b^2$

(ii) Backward method/Reverse method : In this method we will move from conclusion to Hypothesis. Steps are written in opposite/reverse direction. This will be used when the process of moving from Hypothesis to conclusion is difficult.

Ex : a, b are real numbers and $a < b$, then $4ab < (a + b)^2$

Solution :

$$4ab < (a + b)^2 \Rightarrow 4ab < a^2 + 2ab + b^2$$

$$\Rightarrow 0 < a^2 - 2ab + b^2$$

$$\Rightarrow 0 < (a - b)^2$$

$$\Rightarrow a - b \neq 0$$

$$\Rightarrow a \neq b$$

$$\Rightarrow a < b$$

\therefore If $a < b$ then $4ab < (a + b)^2$

2) Indirect method : In this method all alternatives are to be considered to prove the statement. We will show that only one of these alternatives is true and remaining are all false. i.e., conclusion is only remains true

Ex: if two lines are intersected, they intersect at only one point

Here the Conclusion is "two lines intersect at only one point". The alternative for this is "they intersect at many points".

Therefore "two lines intersect at two points" is to be taken and it will be proved that it is a wrong statement. That implies two lines may not intersect at two or more than two points i.e., it is proved that "two lines intersect at only one point". So the statement is proved.

(i) Don't exist natural number x and y such that $x^2 - 4y = 3$

Hypothesis: $x^2 - 4y = 3$

Conclusion: x and y are not natural numbers

Proof: let x and y are natural numbers (alternative for conclusion)

Then x and y may be even or odd

Case (i) : if x is even then

let $x = 2z$

$$\therefore (2z)^2 - 4y = 3 \Rightarrow 4z^2 - 4y = 3 \Rightarrow 4(z^2 - y) = 3$$

So 3 is a multiple of 4. It is wrong/false

Case (ii) if x is odd then

let $x = 2z + 1$

$$(2z + 1)^2 - 4y = 3 \Rightarrow 4z^2 - 4y + 1 - 4y = 3$$

$\Rightarrow 4(z^2 + z - y) = 2 \Rightarrow 2$, is the multiple of 4. It is also wrong or false statement Therefore x is neither even nor odd. So x is not a natural number.

3) Counter Example : Truly speaking it is not a proof. It is logic to prove that the given statement is wrong/false. In this method we will give example which makes the statement wrong

Ex: (i) all prime numbers are odd

For this the counter example is 2

Ex: (ii) there is no biggest prime number

Solution: let p is the biggest prime number then

$(2.3.5.7.11.....p) + 1$ is bigger than the prime number p

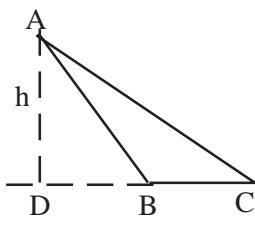
In the class room management, the separatist views are to be noted. For example mathematics is not necessary for a particular group people; girls are not able to learn mathematics. Such faiths will effect class room management. Similarly some unfaihts are there towards some casts. All these are questioned and clarified in the class room.

- SCF 2011

Activity Sheet / Work sheet on proofs

1. Some questions are given below. Write answers in space provided.
2. Answers should be short, clear and reasonable.

	Questions	Answers
1.	<p>This is a 5 words sentences</p> <p>This is not a 5 words sentences</p> <p>Truth values of both sentences</p>	
2.	Definition-Which kind of statement	
3.	The angles which have a common vertex and a common side are called adjacent angles. Is this a correct definition? If not, correct it	
4.	We take the Hypothesis is always true. Why?	
5.	All acids are sour to taste. Liquid 'A' is sour to taste. So liquid 'A' is	
6.	The Gold Back conjecture is not yet proved so it is not a statement. What is your opinion?	
7.	The quadrilateral which has four equal angles is a square- give counter example. Change the statement as a true statement	
8.	At least two persons with same number of hair will exist in the world. How can you say?	
9.	Watson told that-I have been waiting for news paper boy since morning. Sherlock Homes told that -Why are you waiting, today here there is no paper, so the paper boy not yet come. What is the difference between their arguments	
10.	If the capital of Russia is Masco then the capital of India is New Delhi. Here there is no relation between Hypothesis and Conclusion, but it is a true statement. Why?	
11.	This is the month of February. So there are 28days only. Give a counter example	

	Questions	Answers
12.	The equidistant point from the sides of an angle is lie on the line of angle bisector. Draw a diagram and mention Hypothesis and Conclusion.	
13.	Who of the following was awarded with Nobel Prize for two times (1) Neopolien (2) Newton (3) Linnaeus Paling (4) Gaalab	
14.	In triangle ABC, $AC \neq BC$ and in triangle ADC, $AD \neq AB$, then CD bisect the angle ACB- write a flow chart for its proof <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 10px;"> $AC \neq BC$ $AD \neq BD$ $CD \neq CD$ </div> <div style="text-align: center;"> <div style="border: 1px solid black; padding: 5px; margin: 0 auto;"> $\triangle ADC \neq \triangle BDC$ </div> <div style="margin: 5px 0;">↓</div> <div style="border: 1px solid black; padding: 5px; margin: 0 auto;"> $\angle ACD \neq \angle BCD$ </div> </div> </div>	
15.	In a problem on finding the area of a triangle, the adjacent figure is drawn. Write relevant Hypothesis and Conclusion. <div style="text-align: center;">  </div>	

Chapter- 5

APPROACH PAPERS

A) Approach Paper on Number System

A. Necessity :

In natural number system we have the freedom to do addition and multiplication operations only. Always it is not possible to do their reciprocals i.e., subtraction and division. We get the freedom to do subtraction if the negative numbers are added to the natural numbers. To perform division operation we need rational numbers. For the solution of quadratic equations irrational numbers are needed.

B. Fractions :

Normal fraction is an ordered pair of 'a, b'. It can be written in the form of $\frac{a}{b}$. Here $b \neq 0$. 'a' is called as numerator and 'b' is called denominator

Ex : $\frac{2}{3}$, $\frac{4}{3}$ etc.,

Ex : $\frac{x^3 + 1}{x^2 + 1}$, $\frac{x^2 + 3x + 4}{x}$ etc., all these are called algebraic fractions.

Ex: a fraction which is in the form of $\frac{a}{10^n}$ is called as decimal fraction. In this fraction 'a' is an integer and 'n' is a natural number.

C. Rational Number - Definition :

If 'a & b are integers and $b \neq 0$ then the numbers which can be written in the form $\frac{a}{b}$ are called Rational Numbers.

(i) Here b is taken as non zero number, i.e. $b \neq 0$,

(a) If $a = 27$, $b = 3$ then $\frac{a}{b} = \frac{27}{3} = 9$, here b is divisor.

(b) If $a = 27$, $b = 7$ then $\frac{a}{b} = \frac{27}{7} = 3 \frac{2}{7}$. Here generally we consider b as divisor, 23 as dividend, 3 quotient and 2 remainder. In $\frac{a}{b}$, b is divisor. Zero is not considered as divisor so $b \neq 0$.

(ii) Here we noted that "the numbers which can be written in the form....." but not "the numbers which are in the form of". To understand this let us consider the following examples:

Consider $\frac{\sqrt{20}}{\sqrt{5}} = \frac{\sqrt{4} \sqrt{5}}{\sqrt{5}} = \sqrt{4} = 2 = \frac{2}{1}$

Here $\sqrt{20}$ and $\sqrt{5}$, both are not integers. These are not in the form of $\frac{a}{b}$, but we are able to write them in the form of $\frac{a}{b}$. Therefore $\frac{\sqrt{20}}{\sqrt{5}}$ is a rational number.

(iii) Consider $\frac{\sqrt{15}}{\sqrt{5}} = \frac{\sqrt{3} \sqrt{5}}{\sqrt{5}} = \sqrt{3}$. This cannot be written in the form of $\frac{a}{b}$. So it is not a rational number.

(iv) Generally when writing $\frac{a}{b}$, b is considered as positive number.

D. Golden Ratio) :

The number series which starts with $F_1=1$ & $F_2=1$ and progressively continued as $F_{n+2} = F_{n+1} + F_n$ is called Fibonacci numbers. That is $F_1=1, F_2=1, F_3=2, F_4=3, F_5=5, F_6=8$. Rational numbers $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}$ are came from $\frac{F_{n+1}}{F_n}$. Write them in ascending order and decide the place of $\frac{13}{8}$.

If we observe the above ratios (rational numbers) we can note that they are reaching a limit particularly 1.6. this is given in the chapter "Ratios" as Golden Ratio. Again we will discuss it in the chapter "Rational Numbers".

Let us extend the Fibonacci number series. In this series only 144 is the perfect number. Is there any other perfect square number in this series? The answer is not yet found.

E. Middle Numbers :

- The middle number / average of the numbers "a and b" is $\frac{a+b}{2}$. Which lies between 'a and b', i.e., $a < \frac{a+b}{2} < b$. For example the middle number of 6 and 10 is $\frac{6+10}{2} = 8$ and $6 < 8 < 10$.
- Middle number of a and b means it is bigger than 'a' and smaller than "b". For example the numbers 3,4,5,6,7 are lie between 2 and 8
- If we write the numbers between 'a and b' in ascending or descending order then the middle most term is called median of 'a and b'. If $a < x < y < z < b$ then 'y' is median.
- If $\frac{a}{b}, \frac{c}{d}$ are two rational numbers and b,d are positive numbers then $\frac{a+c}{b+d}$ is called the midiant of $\frac{a}{b}$ and $\frac{c}{d}$. Further

In the above, specific principles are given for median and mediant. These are used to write rational numbers, between given any two rational numbers.

Ex: middle term of $\frac{2}{3}$ and $\frac{3}{4}$ is $\frac{\frac{2}{3} + \frac{3}{4}}{2} = \frac{\frac{8+9}{12}}{2} = \frac{17}{24}$ and

mediant is $\frac{2+3}{3+4} = \frac{5}{7}$

if $\frac{a}{b} < \frac{c}{d}$ then $\frac{a+mc}{b+md}$ (m is any number) gives midiants . By changing the value of 'm' we will write as many as rational numbers

Prove that $\frac{a}{b} < \frac{a+mc}{b+md} < \frac{c}{d}$

F. Farey Numbers :

From '0 to 1' means the fractions from 0/1 to 1/1 is called Farey sequence. It is shown by F_n . In these fractions the denominator is always less then 'n'. This is called nth Farey sequence.

Ex: F_1 R $\frac{0}{1}, \frac{1}{1}$

F_2 R $\frac{0}{1}, \frac{1}{2}, \frac{1}{1}$

F_3 R $\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$

F_4 R $\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1$

Write the fraction of F_5 and F_7

F_n is called nth Farey fraction.

G. Terminated, non-terminated, recurring, Non- recurring decimal numbers:

The fractions which are in the form of $\frac{p}{10^n}$ are called decimal fractions.

- (i) If the denominator of decimal fractions is observed it is found that 2 and 5 are factors of the denominator. So if the denominator is in the form of $2^m \cdot 5^n$, then the rational number $\frac{a}{b}$ can be written as terminating decimal number

In $\frac{a}{b}$, if $b = 2^m \cdot 5^n$ and $m \geq n$, then

$$\frac{a}{b} = \frac{a}{2^m \cdot 5^n} = \frac{a \cdot 5^{m-n}}{2^m \cdot 5^n \cdot 5^{m-n}} = \frac{a \cdot 5^{m-n}}{2^m \cdot 5^m} = \frac{a \cdot 5^{m-n}}{10^m}$$

Since $m-n \geq 0$, 5^{m-n} is a integer and $a \cdot 5^{m-n} = c$

So $\frac{a}{b} = \frac{c}{10^m}$

Therefore write 'c' and arrange decimal point according to the value of 'm'

Ex: $\frac{17}{40} = \frac{17}{2^3 \cdot 5} = \frac{17 \cdot 5^2}{2^3 \cdot 5^3} = \frac{17 \cdot 25}{10^3} = \frac{425}{10^3} = 0.425$

Also 40) $\frac{17}{0}$ (0.425

$$\begin{array}{r} 0 \\ \hline 170 \\ 160 \\ \hline 100 \\ 80 \\ \hline 200 \\ 200 \\ \hline 0 \end{array}$$

Apply both the methods and write steps.

- (ii) In $\frac{a}{b}$, if b is not equal to $2^m \cdot 5^n$, then every remainder is one of $1, 2, \dots, b-2, b-1$. So the remainder repeats at one step and from there it will be continued i.e., the previous remainders will be repeated in the same order again and again. So the decimal part of the rational number whose denominator is not in the form of $2^m \cdot 5^n$ is not terminated and repeat again and again

$$\begin{array}{r}
 \text{Also } 7) \quad 10 \quad (1.428571 \\
 \quad \quad \quad 7 \\
 \quad \quad \quad \hline
 \quad \rightarrow 30 \\
 \quad \quad \quad 28 \\
 \quad \quad \quad \hline
 \quad \quad \quad 20 \\
 \quad \quad \quad 14 \\
 \quad \quad \quad \hline
 \quad \quad \quad 60 \\
 \quad \quad \quad 56 \\
 \quad \quad \quad \hline
 \quad \quad \quad 40 \\
 \quad \quad \quad 35 \\
 \quad \quad \quad \hline
 \quad \quad \quad 50 \\
 \quad \quad \quad 40 \\
 \quad \quad \quad \hline
 \quad \quad \quad 10 \\
 \quad \quad \quad 7 \\
 \quad \quad \quad \hline
 \rightarrow 30
 \end{array}$$

The remainder '3' came again i.e., from here the reminders get repeated in the same order.

H. Therefore the rational number which is in the form of $\frac{a}{b}$ can be written in the form of terminating decimal or non-terminating and recurring decimal number and the converse is also holds good.

I. Both are same: we know that

$$1 = 0.9999 \dots \quad R \quad 0.\overline{9}$$

$$0.1 = 0.0999 \dots \quad R \quad 0.0\overline{9}$$

$$0.01 = 0.00999 \dots \quad R \quad 0.00\overline{9}$$

So every terminating decimal can be shown as non-terminating and recurring decimal.

We know that the limit point of a polygon if the number of sides goes on increasing is a circle. But we also know that the number of sides of a polygon is increased for any number it would not be a circle. So generally we may get a doubt that even though the number of 9s is very large in 0.9999..... How is it equal to 1?

If $0.\overline{9}$ and 1 are two different rational numbers there exist other rational numbers between them (Why?), one of them may be their average.

The average of 1, $0.\overline{9} = \frac{1 + 0.9999 \dots}{2} = \frac{1.999 \dots}{2} = 0.999 \dots = 0.\overline{9}$ i.e., we got $0.\overline{9}$ again.

Therefore there is no other rational number between 1 and $0.\overline{9}$

$$\therefore 0.\overline{9} = 1$$

a. Irrational Numbers :

Pythagoras and his followers established a secret institution to find new investigations. They found many new things and kept secret with themselves. Their concept was all numbers are rational

numbers. But the length of the hypotenuse when it was compared with other sides became as irrational number in isosceles right angle triangle. Pythagoreans reacted very seriously on this and kept it secret. If anyone told about it he would be arrested, punished and jailed in a ship. Proclus, who had written a comment on "The Elements", said that: some of the persons among who opened their mouth about the secret of irrational numbers were died in a ship and remaining people were kept in an Island to be beaten by the ways of the ocean.

b. Real Numbers :

All numbers which are used to show the points on the number line are real number i.e., all the numbers on the number line are real numbers

- c.** Let us recall the definition of the rational number. We called that, The numbers which can be written in the form of $\frac{a}{b}$ (a and b are integers and $b \neq 0$) or the decimal numbers which are non-terminating and recurring are rational numbers. That means the numbers which are not rational numbers on the number line are irrational numbers i.e., the numbers which cannot be written in the form of $\frac{a}{b}$ are called irrational numbers. That means the decimal numbers which are non-terminating and non-recurring are irrational numbers.

Ex: 1.01011011101111.....
5.245246247248.....

d. Real number in different numbers systems :

Consider $\frac{1}{7} = 0.\overline{142857}$. Here the number is in base 10 system. Here 142857 repeats .If we change it into base 7 system

$\frac{1}{7_{(10)}} = 0.1_{(7)}$ i.e., it is terminating number.

So if a rational number in one number system is changed into to another number system, still it is a rational number. But an irrational number in one number system is remains an irrational number in any other number system.

$\sqrt{2} = 1.4142135623731.....$ in base 10 number system
= 1.0110101000001001110..... in base 7 number system.

i.e $\sqrt{2}$ is an irrational number in any number system.

e. Union :

$\sqrt{2} + 3$ is it a rational number or Irrational number?

$\sqrt{2} + 3 = a$ a rational number then

$\sqrt{2} = a - 3$ $a-3$ is a rational number (why?)

But $\sqrt{2}$ is an irrational number, therefore $\sqrt{2} + 3$ is an irrational number.

Show that if a is irrational number and b is rational number then $a+b$, $a-b$ are irrational numbers?

$3\sqrt{2}$ is rational or irrational number?

Let $\sqrt{3} \cdot 2 = x$ is a rational number, then

$\sqrt{2} = \frac{x}{3}$ ($\frac{x}{3}$ is rational number, why?)

But $\sqrt{2}$ is irrational so $3\sqrt{2}$ is an irrational number.

Prove that if a, b are irrational and rational numbers respectively then $a \times b, \frac{a}{b}$ are irrational numbers

f. $(\sqrt{a})^2 = a$

$(\sqrt{a})^3 = a\sqrt{a}$, therefore the power of an irrational number is either rational or irrational number.

g. **Why?**

$2\sqrt{2}$ is an irrational number and $(\sqrt{2})^2$ is a rational number. But

$(\sqrt{2\sqrt{2}})^2 = 2$ is a rational number.

So the exponent number of a irrational number is irrational or rational number, the number is either rational or irrational number.

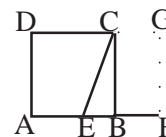
But if we observe $\pi^\pi, 2^\pi, \pi^\sqrt{2}$ they seem to be irrational numbers. But how to find is not yet found.

h. **h. No tired :**

When we discussed about Fibonacci number series, we discussed that the ratio between two consecutive numbers in the series is near to the value 1.6 and this ratio is called Golden Ratio. And in the chapter "Ratio" we discussed that if the ratio of length and breadth of rectangle is in golden ratio then it seems to be very beautiful. Because of this the painters and engineers would make their paintings or plans in the golden rectangular shape. But how is it to be constructed?

Construct square ABCD with side equal to the breadth of required rectangle. Plot the midpoint of AB and name it as E. Plot F on AB such that EC=EF. Construct AFGD on AF.

AFGD is golden rectangle.



Take $EB = 1$, and show that $\frac{AF}{AB} = \frac{\sqrt{5}+1}{2}$. This is also can be written as $\phi = \frac{\sqrt{5}+1}{2}$.

Find its value up to three decimal places. It is an irrational number.

i. In the above golden rectangle AFGD if we remove the square ABCD, then the remaining rectangle is also a golden rectangle. Similarly if we remove the square part from the remaining golden rectangle again, the remaining part (rectangle) is also a golden rectangle. Like this way if we remove the square parts from the golden rectangles then the remaining rectangles are also golden rectangles.

If this process is continued we will get only golden rectangles. So all these are called Dynamic rectangles.

j. **Paper sizes :**

In another context also we can observe the golden rectangles.

The paper which is used in the computer printer is called A_4 size paper. Why?

If the ratio of length and breadth of a rectangular paper whose area is 1 sq.m. is $\sqrt{2}$ then it is called A_0 size paper. If it is halved then the each half is called A_1 size paper. If A_1 size paper is halved then

it is called A_2 size paper. Similarly we will get A_3, A_4 size papers. The ratio of length and breadth of all these different size papers is $\sqrt{2}$. Therefore all these are golden/Dynamic rectangles.

Note that the resulting shapes/sizes in the above two contexts are golden/dynamic rectangles why because the first one is a golden rectangle.

Did you observe anywhere the use of Irrational numbers? -Discuss

k. Adjacent there is no room/space, Are there no neighbours? :

On the number line rational as well as irrational numbers are there. How dense they are means there is no space at all between two numbers i.e., the number line is full of with real numbers, for every number on the number line on both sides there are numbers. That means we cannot say the adjacent number for any number on the number line. Discuss why this is happened?

l. Probability :

In our discussion points this is the last one. On the number line there is infinite number of rational and irrational numbers. If we put sharp edge of a pencil at a point on the number line, then it is too difficult to tell whether the point represents rational number or irrational number.

But the following experiment trays to give answer

Take a dice of 10 faces and write the numbers from 0 to 9 on its faces (one number on each face). Roll the dice, observe the number on its face, write the same number after decimal point (take its integral part as zero). Roll again the dice, observe again the number on the dice, and write this number in the second place. If it is continued, find whether the resulting number is rational number or irrational number.

Indian arts and cultural rangoli are not only beautiful to look at, but there is much mathematics to learn.

- SCF 2011

B) Approach paper on Algebra

Importance :

1. Calendar:

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

- ◆ Take any 3 x 3 square in the above table and find the sum of the numbers in it
- ◆ Find the least number in the same 3 x 3 square
- ◆ Add 8 to the least number and multiply the sum by 9
- ◆ Compare the first and second results
- ◆ Can you explain the secret of this puzzle with the help of Algebra?

2. Pythagorean Triples :

- ◆ Take any two consecutive even numbers (Ex:- 2 and 4)
- ◆ Find the sum of their reciprocals (Ex:- $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$)
- ◆ Consider numerator, denominator, denominator+1 from the above resultant fraction (Ex:- 3,4, 4+1=3,4,5)
- ◆ Check whether the resultant numbers are Pythagorean triples
- ◆ Check whether it is true for any two consecutive even numbers
- ◆ In this process which branch of mathematics is used?

3. Magic Squares :

8	1	6
3	5	7
4	9	2

3 x 3 magic square

4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

4 x 4 magic square

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

5 x 5 magic square

Above are 3 x 3, 4 x 4, 5 x 5 magic squares. When these are filled with first 'n' natural numbers i.e., 3 x 3 magic square is filled with 1 to 9 numbers, 4 x 4 magic square is filled with 1 to 16 numbers then the constant (Sum of the numbers in a row/column) are given below

Type of magic square	3 x 3	4 x 4	5 x 5
Constant	15	34	65

From the above table can you guess the constant of 6 x 6 magic square?

Let us observe the above table in detail once again

Type of magic square	3 x 3	4 x 4	5 x 5
Constant	$15 = 3 \times 5$ $= \frac{3 \times 10}{2}$ $= \frac{3(3^2 + 1)}{2}$	$34 = \frac{4 \times 17}{2}$ $= \frac{4(4^2 + 1)}{2}$	$65 = 5 \times 13$ $= \frac{5 \times 26}{2}$ $= \frac{5(5^2 + 1)}{2}$

Hope that you understood the relation in the above pattern. If we proceed in the same way then we will get the constant of 6 x 6 magic square as $\frac{6(6^2 + 1)}{2} = 111$.

Can you generalize this relation with the help of Algebra? And can you check it?

4. Prove the following :

- ◆ Show that the sum of a number of four digits and the number formed by reversing the digits is always divisible by 11.
- ◆ The greatest and least of four consecutive numbers are multiplied together; so also are the middle pair. Show that the difference of the two products is always 2.
- ◆ The sum of two consecutive numbers is always an odd number.
- ◆ Which branch of mathematics is used in the above proofs?

Then do you agree the following :

- 1) Algebra is a powerful branch of mathematics to solve puzzles and problems.
- 2) Algebra is very helpful in examining the rules/laws.
- 3) Algebra is used in generalization of laws/rules.
- 4) Algebra is used to generalize a pattern.
- 5) Algebra is used in finding an unknown value/quantity.

In this way the knowledge and understanding of algebra helps not only to solve the puzzles and daily life problems but also helps for the development of mathematics.

If we don't learn algebra we cannot understand many ideas discussed in Chemistry, Physics, Earth science, Economics, business, Psychology and other areas. Lack of knowledge in Algebra limits our opportunities. Without algebra it is not possible to do many things as easily as at present and it is even not possible to find/discover/invent some results.

If we want to explain anything for one time there is no need of algebra. But we want to explain anything for more than one time (More times) then we need algebra. Algebra is best mathematical language to explain such things very easily. For example if we want to explain how to find the product of two fractions, we have to write in the following way:

"Multiply the numerator of the first fraction with the numerator of the second fraction, and denominator of the first fraction with the denominator of the second fraction, then write the results as numerator and denominator respectively."

But by using the algebra we can write it very easily in the following way:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

It is very handy and it can be understood by each and every one. If we are able to explain the uses of algebra, in this way, then students may get inspired to learn algebra.

Algebra - Application :

- Formulas :** Once a news paper in Chicago, in its sports column published the following news item.

"Competitions will be held within 3 weeks, but the pre-competition game to get all teams ready, which is going to be held on Friday night, is as important as the **formulas** for high school student, so watch it without fail".

From the above sentence we can understand the importance of the formulas in mathematics. Further we can understand that the writer also knows about how far the application of algebra is needed in sports and games. Really speaking many of the issues pertaining to the sports and games have a link with formulas. For example if a player gets T points in ϵ_1 games, then his/her average score is $\frac{\epsilon_1}{T}$. Similarly we can estimate a player's individual capacity by taking his/her maximum and minimum score (range) into consideration, further we can calculate the winning percentage of a team with the formula $\frac{W}{W+L} \times 100$, where W = number of wins, L = number of losses.

Similarly to know the area of our residential flat, to know how much cloth is required for our shirts, to know how much length of fence is required to fence our house, to know how much length of ribbon is required to pack a gift that will be given to our friends, in the calculation of rebate, tax, sales tax, income tax and in every moment of our life, we use formulas. We use very simple formulas to calculate them. But all formulas, like above are not so simple. For example to know what date falls on which day the formula is

$$W = d + 2m + \left\lfloor \frac{3(m+1)}{5} \right\rfloor + y + \left\lfloor \frac{y}{4} \right\rfloor - \left\lfloor \frac{y}{100} \right\rfloor + \left\lfloor \frac{y}{400} \right\rfloor + 2$$

Here D = day of the date

M = month of the date,

Y = year of the date

But when we are taking the value of month we have to take the value of January as 13, February as 14, and the value of remaining months as it is i.e., the value of the month march is 3, April is 4 December is 12. And [] means we have to take integral value only

i.e., $[5.2] = 5$, $[14.75] = 14$.

Once "w" is computed divide 'w' by '7' and as per the remainder we find the day of the date. If remainder is zero, the day is Saturday; remainder is 1, the day is Sunday and so on with Friday = 6.

Similarly we can compute the value of monthly installment when the amount "A" is borrowed with rate of interest 'r', for 'm' months from the following formula.

$$P = A x^m \left(\frac{x - 1}{x^m - 1} \right)$$

where $x = 1 + \frac{r}{1200}$ here 'p' is monthly installment.

According this formula if ₹.8500/- is borrowed with 11.25% rate of interest for 4 years the monthly installment that is to be paid is ₹. 220.72

2) **Functions** : the following are the examples of functions.

How the various health factors will change with respect to age? How the increase in weight effects on health conditions of person,? How a family budget will effect, when their spending habit is changed, how the growth of population effects on use of different resources/ energies on the earth.

Without using algebra/functions, we can also able to explain them with large tables and graphs. But we cannot get as much clarity, simplicity as when algebra is used. Further with the help of algebra we can find when they have maximum value and minimum value. All these are foundations for calculus.

3) **Linear Equations** :

Anything that changes at a constant rate gives raise a linear equation of the form $T = Ax + By$. By using such equations how the real life problems are to be solved is given in the 8th class text book. Similarly how to find the cost of renting a car; the cost of renting a book from a library etc., is given the new 9th class text book.

4) **Slope** :

Any quantity that changes must change at some rate. That rate of changes is often very important. The change in car's velocity affects its acceleration. The change in our earnings affects our financial status. The change in the status of unemployment will affect inflation. The algebraic idea behind all these notions is "slope".

5) Exponents :

The branch of mathematics which is developed to estimate the possibilities of an incident is 'probability'. Now exponents are being used in this branch. If any quantity that changes at a constant growth rate, that growth is called exponential growth. We can observe such growth in our daily life. Compound interest, mortgage value of a vehicle, payments on credit cards, payments of life insurance policies, benefits on retirement... all are belong to this category. The same mathematical concept is used in predicting growth in population, animals etc.

6) Quadratic expressions :

According to Newton "the path of any object, under the influence of gravity, will be explained by the quadratic expression $Ax^2+Bxy+Cy^2+Dx+Ey+F$. The same is applicable to the stars, planets, comets and moons. Similarly we can note that the same is also applicable to the paths of rockets, bullets, base ball, basket balls. That means this concept is used in physics, space science, armed services, games and sports.

7) Logarithms :

These are used in rector scale used to measure the intensity of earthquake, Ph scale used to measure the acidity, the star magnitude scale used in astronomy.

8) Permutations and combinations :

To predict the result in lotteries, to determine number of people that needed to conduct opinion polls, to predict the T.V. ratings, these permutations and combinations are used.

Like this way algebra is very important branch of mathematics. If we visit a place, but don't know their language, we can any way able to get along by using signs/signals. But we cannot understand their social, political and cultural values and status; we cannot even able to appreciate the richness of their culture. And the pity thing is we even cannot understand what we missed in this visit. Similarly we can learn mathematics without algebra, but we cannot able even to appreciate its beauty, structure and mystery.

History :

generally we feel that arithmetic taught in lower classes is very easy and the algebra taught in higher classes is difficult. The reason, probably, behind it is arithmetic deals with very familiar symbols like digits/numbers, where as algebra deals with unfamiliar symbols like x, y, \dots . But practically there is no difference in using symbols like digits/numbers and symbols like x, y, \dots

In earlier day of civilization the following was to be used.

" If you have six fruits and if I give you five more same fruits, then how many fruits do you have all together?"

we know that eleven fruits is answer for this question. i.e.,

six fruits + five fruits = eleven fruits

But if the numbers are very large, writing in the above method is very tedious and time consume. As a remedy for this problems, 1,,2 ... 9 were developed. These are called numerals.

We already learnt how to write bigger numbers with these numerals. Because of this it is possible to write bigger sentences like "six thousand seven hundred and fifty two" simply as "6752".

By all means you understood the use of signs or symbols. In our daily life we use practically all such symbols for example we use the symbol "horse" to represent a particular life on the earth. Like this way in our daily life to express anything very easily, we use signs/symbols. We learnt all these symbols in our earlier age in lower classes and we got sufficient training. Due to this we felt that all these are very easy. But if we want to use new signs/symbols in higher classes we will hesitate somewhat. We naturally do not like such new symbols to use. But remember that when we were learning the new symbols like 1, 2, ..., 9, +, - in lower classes we would have faced the same problem. Any how it is clear that the use of symbols will simplify our life style.

Observe the following questions.

- 1) If two is added to two then what is sum?
- 2) If five is subtracted from eight what is the result?

By using symbols these two sentences can be written as in the following:

- 1) $2 + 2 =$
- 2) $8 - 5 =$

But when we changed them into symbolic from we would unable to write symbols for total sum in the 1st sentence and for the result in the 2nd sentence. That means we need some other new symbols. But some of us may feel that why we need new symbols, what is wrong if we leave an empty place instead of using symbols. Let us observe the following example:

How much number of mangoes is added by six mangoes to get eleven mangoes? Here we don't know at the beginning how many mangoes are there? This is an unknown quantity. If we write the above sentence by leaving empty space for this unknown quantity, it is like the following:

$$6 = 10 \rightarrow (1)$$

It looks funny and is hard to read. So it is not the better thing to leave blank space for unknown. How the symbol is 6 for six, 10 for ten, and '+' for addition we also need a symbol for unknown quantity.

It is felt that you understood the importance and need of symbols. If we are thinking that using symbols for unknown that means we are thinking about algebra. If a symbol is made for unknown then we will try to use this symbol for all unknowns in arithmetic. That is why algebra is called as generalized arithmetic. Now our problem is to find a symbol for this unknown. For this purpose many symbols were examined. When they were examined some issues were kept in mind. They are, it should be

familiar to all, it should be handy, easy to spell/speak, and it should be least frequently used in its original place. The symbols with such characters are x, y, z Of English alphabet.

By using "x" for unknown in the above equation (1) it seems to be in the following:

$$x + 6 = 10 \rightarrow (2)$$

By observing (1) and (2) we find that (2) is easy to understand. Then

**Do you agree with the following statement?
"Symbols make life simple".**

The first person to use symbols for unknown was a French mathematician named Francois Vieta.

Variable :

Pavani, and Sagar were drawing the models of cars. They use block stickers for the wheels. Pavani made the following model of car with two block stickers.



Then Sagar with the help of two more block stickers made another model of car and put it next to the first model as in the following figure



Mean while their friend Ravi joined them. He asked them "how many block stickers will be required to make four such 4 cars." Immediately Pavani counts the number of block stickers in the above two models doubles the number and says 8.

"Well" Ravi said and asks them, "How many block stickes will be required to make such 59 cars. Pavani and Sagar feel this method of counting stickers Is a bit laborious and time consuming, especially when the number of faces are very large. They decide to find a new way. They think a while and make the following table.

Number of Cars	1	2	3	4	...
Number of required block stickers	2	4	6	8	...

Do you deduct any relation from the above table? The same relation was written by Pavani and sagar in the following way:

$$\text{Number of required block stickers} = 2 \times \text{number of required cars} \rightarrow (1)$$

From this relation Pavani and Sagar are able to find the number of required block stickers to make 59 such care models. To understand easily, if we let the number of required car modes is "x" then

Number of required block stickers = $2x$.

If we want to make a car model then the value of $x = 1$. So the number of required block stickers = 2

Similarly if we want to make two car models then the value of $x = 2$. So the number of required block stickers = $2 \times 2 = 4$

Like this way we can find the number of required block stickers to make the number of required car models.

Here 'x' is called variable. It takes the values 1 or 2 or 3 That means the value of 'x' is not fixed. Its value changes accordingly.



Give some examples of where we use variables in our daily life.

E. Approach paper on Geometry

Importance of study of Geometry :

Geometry is a branch of mathematics concerned with questions of shape, size, relative position of figures and the properties of space. Geometry arose independently in a number of early cultures as a body of practical knowledge concerning lengths, area and volumes, with elements of a formal mathematical science emerging in the west as early as Thales (600 BC). By the 3rd century BC geometry was put into an axiomatic form by Euclid (Greece), whose system considered as Euclidean Geometry set a standards for many centuries to follow. Euclid's collection of geometrical treatise "The elements" had given world wide changes in the study of geometry. This book has been considered as most popular and most translated book into other languages other than Bible.

The study of Geometry laid foundation to the entire educational system in western countries and created logical thinking process among you this from Euclidean time. Its study has been sidened to solve many problems after non-Euclidean geometry came into exist. Scientists, Engineers used many geometrical applications to find new relations and developed wonderful constructions like dams, bridges, roads etc. The art and culture has been developed further and painters, designers and skilled persons created wonderful arts and designs through 3D figures.

For example, the present cinema screen, TV screen were naturally the application of Euclidean geometry only. But it is unfortunate that this kind of applications were not given importance in secondary mathematics curriculum.

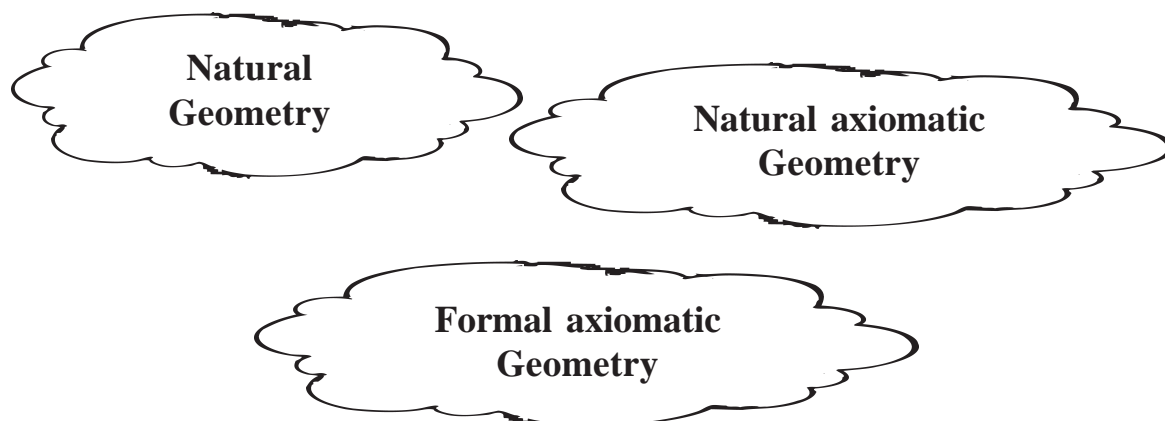
Geometry is derived from two Greek words "Geo" means earth and "metron" means measurement. Ancient Greeks arranged the concepts and axioms according to the logical order. There are certain evidences about circumference of a circle is divided into 360 equal points by Babylonians, right angled triangle theorem is used before Pythagoras and ratio proportional problems, properties of triangles and relation between sides and angles of triangle.

The study of Geometry gives many answers to question raised from Babylonians to Egyptians. All these questions were begin with "How?" For example, how can you find the interest on an amount borrowed? How can we calculate the volume of a pyramid? How can we find the square root of a given number etc were given for reasoning raised many logical thoughts. The importance of Economics increased since 100AD in Western and middle East lay the foundation to learn mathematical language. The mathematical questions were changed according to the new thinking process and begin with "Why". For example why the area of triangle is half the product of their base and height? 'Why' the square on the hypotenuse is equal to the squares of sum of two sides

of right angled triangle? This type of questions promoted many people to think logically with reasoning. The revolutionary approach leads to many developments in Science and thechnology and Trade and comerce. Though the study of geometry undergone many changes the foundations laid by Greeks still influences the society.

How to learn Geometry? :

We can divide the learning of Geometry in three stages.



"**Natural Geometry**" is intimately related to reality and reasoning is close to experience and intuition. The objects of this geometry are material objects, graphic livges on a paper sheet or virtual lines on a computer screen. Drawing and measurement techniques with oridinary geometrical tools (rules, set square, compass) as well as experimentation in the sensible world (using techniques such as paper folding, superposing) are used in this stage. New knowledge may be produced based on evidence, experience or reasoning, while a permanent motion between the model and the reality enables the student to 'prove' the assertions. The kind of experiences were given in 6 and 7th class new text books.

"**Natural axiomatic Geometry**" deals with geometrical figures in which the new relations were obtained through investigations. The objects are ideal, so reasoning relies on the mathematical properties of the abstract geometric objects. a system of definitions and axioms is necessary for the creation of the objects. In this system the axioms are as close as possible to intivition, but making progress and reaching certainty demands demonstrations inside the system. Hypothetical deductive laws are the source of validation. The topics covered in class 9th and 10th were based on certain geometrical relations obtained by the sutdents through verification and activities. For example, to prove "The diagonal divides the parallelogram into two congruent triangles", the child should verify the fact by drawing a parallelogram with suitable measurements and by cutting it into two pents through diagonal and verify whether they coincide eachother or not. He should also know certain axious related to congruency of triangles. this leads the child to prove the theorem using axiomatic approach.

"Formal axiomatic Geometry" discuss the concrete ideas of geometry in symbolic form. The lines, curves, triangles, circles will be translated as algebraic equations. This leads to discover new relations in engineering and Technology. For example $ax + b = c$ ($a \neq 0$) is a linear equation in one variable, which represents a straight line whereas the quadratic equation $ax^2 + bx + c = 0$ ($a \neq 0$) represents a curve.

The passage from natural Geometry to axiomatic natural geometry is a complex, sensitive and crucial matter. Since these two paradigms are different as far as objects, techniques and validation mode are concerned. Moving from Natural Geometry to axiomatic Geometry students have to change their theory concerning the nature of the objects and of the space. They are forced to adopt the notion of conceptual objects, the existence of which is based on a definition in an axiomatic system. Consequently, they have to foster new techniques to work relying on the mathematical properties of each abstract geometrical figure. For example, as a consequence of the continuous practice of the pythagoras' theorem and the didactical contract formed during teaching, students consider that they are expected to apply pythagoras' theorem anytime a right angle is involved in a geometrical figure.

Discussion points

Discuss and write.

- 1) Make a list of some objects you will find at home and write geometrical concepts involved in them.
- 2) Give two examples each in which children learn Geometrical concepts through (a) objects (b) mathematical instruments (c) Experiments (d) logical reasons.
- 3) Explain method of proving the statement "Sum of three angles of a triangle is 180° ".

Geometrical constructions - A Special approach :

Geometrical constructions play a vital role in the Study of Geometry. These constructions will help the children to understand the accuracy and logic behind the geometrical figures. It is a very interesting feeling of persons of different age groups while drawing or constructing figures. It is a challenge to the students. If they learn the basic geometrical constructions in the beginning, it will certainly help them to construct difficult constructions later.

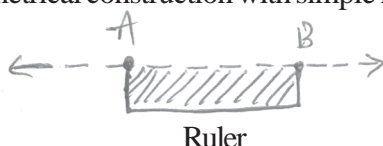
Practicing the basic geometrical constructions and using them in meaningful situation, using proper language to express the construction, choosing proper instruments will lay a foundation to understand abstract concepts behind Geometry. These are the pillars of construction of knowledge in Geometry.

In the development of Geometrical constructions there is a need of Justification through axioms at secondary level. For any basic geometrical constructions, there is a rule to use two primary instruments a ruler and a compass only. But recently we are using mostly graded scale to draw line segments and protractor to construct angles. Though these instruments will help the children to do

the construction with ease, there we are neglecting the national inner abilities of the children, and hence they are not enjoying the beauty of geometrical constructions.

'Ruler' is used to draw a line segment between two given points and 'compass' is used to draw an arc or circle when a fixed point and given a radius. Some people argue that all line segments and angles can not be made by the two instruments.

We can not trisect all angles, but Greek people used these two primary instruments to prove any geometrical construction with simple logic.



Ruler



Compass

Generally, we observe two types of questions appear in text books in Geometrical constructions

- 1) Draw the figure using the given measurements
- 2) Construct the figure using the given measurements.

- ◆ Here for drawing Geometrical figures, we can use any available instruments, whereas to construct the figure, we have to use ruler and compass only.
- ◆ We can see the geometrical constructions given in the new textbooks were different from earlier books. To make the children understand better way of constructing the geometrical figure, figures were given for each step and motivated the children to use ruler and compass only. The children have to use protractor for verification of the angle in the figure and not for constructing an specific angle. The systematic algorithms given in new textbooks will help the children to understand the logical reasoning behind the constructions.

Discussion Points

Discuss and write.

- 1) Construct an equilateral triangle using ruler and compass. Write steps of construction.
- 2) Construct the following angles using ruler and compass and verify using protractor.
 - a) 45°
 - b) 90°
 - c) 135°
 - d) 180°
- 3) Explain how you can trisect 90° angle.
- 4) Construct and verify the Euclid's parallel axiom (Modified by John Playfair). "Through a point not on a given line, exactly one parallel line may be drawn to the given line" using ruler and compass.

Reference Books :

- The Geometry reasoning of Primary and Secondary school students - George panouras, Dept. of Education, University of Cyprus.
- Position paper "Teaching of Mathematics" National curriculum Framework 2005, New Delhi.
- Position paper "Teaching of Mathematics" State Curriculum framework-2011, A.P.

F) Approach Paper on Statistics

What is Statistics?

Statistics is a branch of Mathematics which deals about collection of information, scientific analysis of the data, interpreting and drawing inferences from the data, making estimations, . . etc. It also discusses about the pros and cons of previous projects and makes future plans.

Why do we study Statistics?

In every field of life, such as business, medicine, scientific research, administration, economics, . . etc., it became a routine to collect, analyse and interpret the numeric data, compare and contrast two similar data and to plan for future projects. On par with scientific advancement and population growth, it is inevitable to collect and analyse the numeric data. Many intellectuals are required to work upon. Here statistics helps to fulfil the needs of the globe.

Every individual is benefited when he / she study numeric data and able to make out inferences by scientific reasoning. Every information centre of every department exhibits the information in the form of tables and graphs. An individual who have the knowledge of these can only be benefited. So it is inevitable for every social being to learn statistics.

The sexy job in the next ten years will be statisticians. Because now we really do have essentially free and ubiquitous data and the complimentary scarce factor is the ability to understand that data and extract value from it.

- -

Hal Varian, Google's chief economist(2009)

Evolution and History of Statistics :

Many believes that statistics has evolved as a subject in 1662, when the book, 'Natural and Political Observation upon the Bills of Mortality', written by John Graunt was published first. Initially scope of statistics increased with the use of statistical methods to analyse the religious, demographic and economical data. Now it is not a wonder that there is no field in the world, which is not using the statistics for its progress. Development of computerisation made the usage of statistics intensive.

What are we going to study in statistics during Secondary Education?

On the basis of level of comprehension of the children, the following are discussed step by step from VI to X classes.

- Collection of small Data
- Organisation of Data
- Interpretation and representation of data in terms of Picto graphs, Bar graphs, pie charts, . . etc.
- Constructing Frequency Distribution tables
- Determining the values of Central Tendency
- Drawing inferences and Commenting on Data.

What is a Data? Why it should be organised?

Collection of numeric or any other information, collected in direct survey for a particular purpose or collected from some other data, is called 'Data'

If there are limited number of observations, it is easy to analyse and interpret the data, but if there are many number of observations it is difficult. Then interpretation can be made easy by organising the data in a table form or by constructing frequency distribution. This makes the data concise and easy to handle, but interpretation becomes approximate.

Example 1:

Primary data : Means of transport in each of students houses are listed as

Anitha Auto

Bharathi bullock cart

Chandrasekhar bicycle

Damodar scooter, bicycle

Fareed bullock cart, bicycle

Giridhar bike, auto

Peter bike

Sanjana none

Salma bicycle, auto

Umesh bullock cart

Organised data :

		Means of transport	Number available
		Bullock carts	3
		Bicycles	4
		Scooters	1
		Bikes	2
		Autos	3

When there are limited number of observations, recording the observations, comparing themselves and comparing with other data and interpreting the inferences is simple, but when the observations are many it is possible only when frequency distribution is constructed.

Ex: Consider the marks of 50 students in Mathematics in Quarterly examination

Primary data :

31, 14, 0, 12, 20, 23, 26, 36
33, 41, 37, 25, 22, 14, 3, 25
27, 34, 38, 43, 32, 22, 28, 18
7, 21, 20, 35, 36, 45, 9, 19
29, 25, 33, 47, 35, 38, 25, 34
38, 24, 39, 1, 10, 24, 27, 25
18, 8

Ascending order of the data:

0, 1, 3, 7, 8, 9, 10, 12, 14, 14, 18, 18, 19,
20, 20, 21, 22, 22, 23, 24, 24, 25, 25, 25, 25, 25, 26,
27, 27, 28, 29, 31, 32, 33, 33, 34, 34, 35, 35, 36, 36,
37, 38, 38, 38, 39, 41, 43, 45, 47.

Class Intervals (Marks)	Tally Marks	Frequency (No of students)
0 - 7		4
8 - 15		6
16 - 23		9
24 - 31		13
32 - 39		14
40 - 47		4

Grouped frequency distribution tables is shown aside

If we observe the above three forms Frequency distribution seems to be more concise and comprehensive. So we practically use the frequency distribution of a data.

Let us find out how easy is to interpret a data of limited observations.

Example 3: Price(in rupees) of a kilogram of rice at different shops is 22, 35, 37, 42, 24, 35, 41, 37, 40, 37, 30, 39, 42, 38, 37, 41, 37.

Ascending order of prices is : 22, 24, 30, 35, 35, 37, 37, 37, 37, 37, 37, 38, 39, 40, 41, 41, 42, 42.

- ♦ Minimum value of observations is 22; Maximum value is 42; difference of these is $42-22=20$. This is called 'Range' of the data. It represents the spread of observations in the data.
- ♦ Out of 17 observations, the middle most observation is 'Price of a kilogram of rice is ₹ 37'. It means in how many shops the price of rice is more than ₹ 37, in same number of shops the price is higher than ₹ 37. This value is called 'Median' of the data.
- ♦ By observing the data we can find that ₹ 37 repeats for more number of times. It means in many number of shops price of kilogram of rice is ₹ 37. This is called 'Mode' of the data.
- ♦ Similarly by dividing the sum of observations by number of observations to get 'Average' or 'Arithmetic mean' of the data.
- ♦ Among the above Average / Arithmetic mean, Median, Mode lie at the centre of organised data and so these are called 'Values of Central Tendency'.

When to use which value of Central Tendency?

In classes VI and VII, determination of values of central tendency is discussed comprehensively. But a particular value of central tendency is required to interpret a particular data. This is to be taken care while explaining and drilling. More exercises are given in VII class for the purpose.

How to Construct a Frequency Distribution Table?

Out of many observations of a data, if there are few distinct observations, then the data can be organised in frequency distribution table (as in example 1). This is called 'Ungrouped Frequency Distribution'.

If there are too many distinct observations, ungrouped frequency distribution of the data cannot be concise or easy to read. Then ascending or descending order of observations can be divided into convenient groups and observations can be noted as frequencies. This is called 'Grouped Frequency Distribution'. When to use Ungrouped or Grouped data is discussed with examples in VIII class. Make distinction.

Determination of values of central tendency of (i) primary data is discussed in VII class and (ii) ungrouped frequency distributions is discussed in IX, where as (iii) grouped frequency distributions is discussed in X class.

Graphs for Comprehensive Understanding:

Graphical representation of a data is more understandable rather than in tabular form. When the observations of data are not inter-dependent and distinct observations are few with frequencies we use to represent the data with pictograph. These can be interpreted even by small children and illiterates. So we use them right from the beginning of study of statistics, i.e., from III class onwards. But construction of these is time consuming and so, from VI class onwards, we introduce 'Bar graphs' instead.

In bar graphs different observations are represented by bars of equal width and lengths proportional to their frequencies. As all the distinct observations are independent from one another series of bars can be altered for convenience. When the frequencies of different observations are differ by small extent the data is shown in Vertical Bar Graph and when they differ by much Horizontal Bar Graph is used (you may guess 'why?')

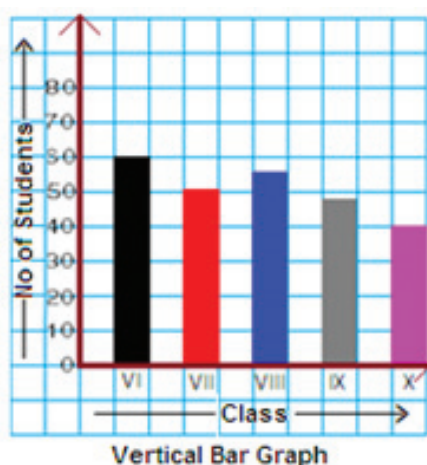


Fig. 3

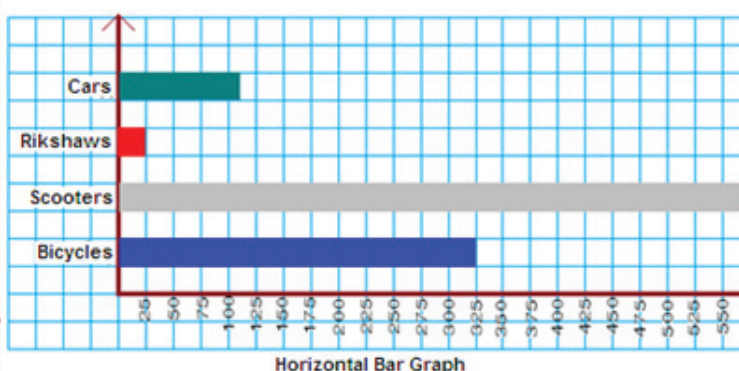


Fig. 4

Which of the following data are to be shown as vertical bar graphs and which are horizontal bar graphs? Discuss.

- 1) Data about the number of boys, girls, youth, old aged people in a colony.
- 2) Stock in a fair price shop
- 3) Production of different brands of cars in an year in India.
- 4) Ingredients used in preparation of a South Indian Dish.
- 5) Rainfall in a place during successive years.

As the understanding level of children is increasing 'Double Bar Graph' is introduced in VII class. If all the distinct observations of a data are considered as parts of something and they are interdependent 'Pie Chart' is used. In this chart angle of each sector is proportional to fraction of each observation in the whole, keeping the sum of the angles is 360°.

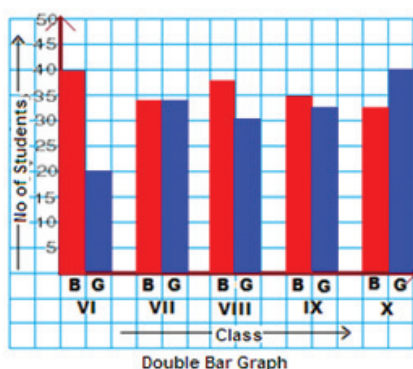


Fig. 5



Pie Chart

Fig.6

What is the difference between a Bar graph and a Pie Chart?

In a bar graph lengths of bars are compared to interpret the data, but lengths of different bars are not inter dependent. In case of a pie chart, each sector represents an observation as how much part of the data it is. (Area of each sector is to area of the circle is as to the quantity of the observation to the whole data). Change in the quantity of every observation affects the fraction of the other observations.

Which of the following data are to be shown as bar graphs and which are pie charts? Discuss.

- 1) Yield of different crops in a village.
- 2) Achievement of a student in different subjects.
- 3) Allocation of funds to different expenditures from the salary of person (Monthly budget of a salaried employee)
- 4) Allocation of funds to different fields in Andhra Pradesh State Budget.
- 5) Number of students of each class in a school.

Up to this point we have discussed about graphical representation of raw data or a data of observations with frequencies (ungrouped frequency distributions). Now we discuss about the graphs of grouped frequency distributions.

Graphs to draw inferences from a data :

To represent grouped frequency distributions we use graphs like 'Histogram', 'Frequency Polygon', 'Frequency Curve, etc. Though the data is same represented in the above three graphs with class

intervals / mid values on X-axis and corresponding frequencies on Y-axis, successively their accuracy increases in drawing inferences. This is explained in VIII class.

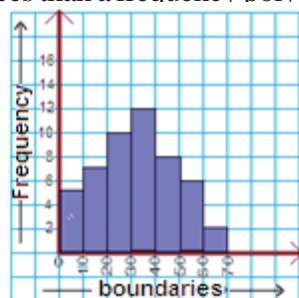
In histogram all the vertical bars similar observations in different intervals and bars are touching one another. As the bars represent the frequencies in successive intervals position of bars should not be altered. Area of all the bars represent the number of observations in the data. This graph is useful in finding the mode of the data.

In frequency polygon also area sirscribed by the polygon with X-axis represents the number of observations in the data. It is useful to estimate the frequency of any observation in the data.

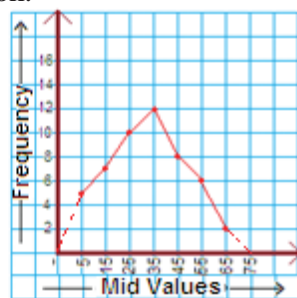
Frequency curve is also useful to estimate the frequency of any observation in the data. But it gives more accurate inferences than a frequency polygon.

Class Intervals	Frequency
0 – 10	5
10 – 20	7
20 – 30	10
30 – 40	12
40 – 50	8
50 – 60	6
60 – 70	2

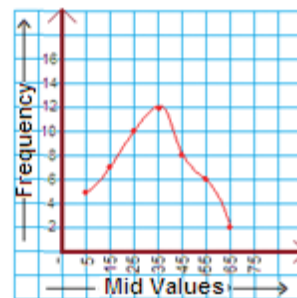
Frequency Distribution



Histogram



Frequency Polygon



Frequency Curve

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Other types of graphs used to represent grouped frequency distribution are 'Ozive Curves' (Less than cumulative and greater than cumulative curves). These graphs are useful to find out number of observations of the data which are greater than or less than a particular observation. Slope of these curves represents the rate of rise or fall of observations at a particular observation.

Class Intervals	Frequency	Less than Cum frequency	Greater than Cum frequency
0 – 10	5	5	50
10 – 20	7	12	45
20 – 30	10	22	38
30 – 40	12	34	28
40 – 50	8	42	16
50 – 60	6	48	8
60 – 70	2	50	2

Cumulative Frequency Distribution

Fig.11

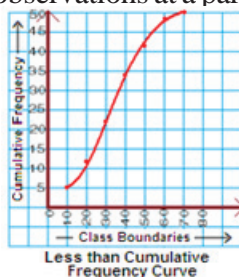


Fig.12

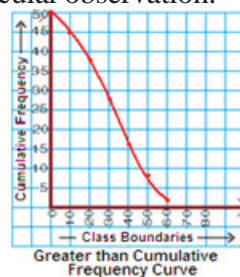


Fig.13

When both the ozive curves of a data are plotted on the same graph their intersection represents the Median of the data. All about the graphs is comprehensively discussed in VIII class. Teachers must reinforce these ideas in IX and X classes.

Note :

There are some exceptions in statistics. For example, values of central tendency of a raw data and its frequency distribution are slightly different, mode of frequency distribution is supposed to lie in the class interval of the higher frequency, but it may not be actually. Use of all such anomalies are discussed the text books. For additional details go through the text books and make every student to read the text books.

G. Approach paper on Mensuration

The primary needs of human life are food, cloth and shelter. To find the measure of these needs we require basic measurements. To find the volume of vessel to store the food, to find the area of place required to construct a house, to find the length of cloth for making a dress we need the concept of area and volume. The topics related to these areas are kept as lessons in 7,8 and 9th classes in our new textbooks. We have introduced the concepts and various problem solving regarding areas of rectangle, square, triangle and circle in 7th class. Number of examples were given related to daily life situations in which we require area and perimeter.

We have introduced "Areas of plane figures" in class 8th as a chapter in which Wide variety of problems given to find the area of land required to construct a house when the land is in the shape of a trapezium. Through these examples the formula to find the area of trapezium is derived. In the same way we got this formula by an activity in which a trapezium was drawn on a graph sheet and folding along the diagonal to create triangles. As we know the area of triangle in earlier class, we got the area of trapezium by adding the areas of two triangles. Here you can observe that we have followed the learning principle "known to un known". Another way to find the area of trapezium was also given by dividing the trapezium in to a rectangle and two triangles.(see Area of play ground problem).In all these three cases we have generalised the principle to find the area of trapezium. To reinforce the concept and formula we have been given some illustrated examples.

Activities were provided to children to find area of quadrilateral by triangulation method under Try these as the children were exposed to find the area of any quadrilateral by the previous activities. Some problems were also given to find area of rhombus by giving examples from daily life situations.

Many villagers doesn't know how the surveyors measure the lands which are in different shapes. A surveyor measures the lands using the concept trapezium, rectangle, triangle.

A circle is not made up of line segments. we have used a graph paper to find it's area. But it doesn't give exact area. so an activity was given to find the area of circle by dividing it in to 8 equal parts in which each part as a sector and arranged in an order to get a shape of a rectangle or a parallelogram..But

to find the area more accurately we need to divide it in to 64 equal sectors. This formula has been derived by doing an activity given in the book "A book of juice". We have also provided the topics like area of semi circle, area of rectangular paths with illustrated examples to practice for competitive examinations at national level.

We have introduced the surface area and volumes of cube and cuboid through 2d and 3d figures. As the children were more exposed to 3D and 2D geometrical figures and relation between them topics were given to find surface area of cylinder, cone and sphere in this chapter. Children were provided with activities to find the surface area and volume of these objects in detailed manner. Raised questions to think and discuss how the capacity is different from volume of a vessel. The volume of cylinder is derived through area of a circle(base) and multiplied with it's corresponding height. Relation was established between the volume of cone and a cylinder by an experiment as the volume of cone is $1/3^{\text{rd}}$ of volume of cylinder. The volume of sphere was derived through the volume of pyramid.(see the figure and activity given in the textbook)

Children were provided with more conceptual ideas through logical questions. How a prism is different with pyramid? Are cube and cuboid are prisms? Why a cylinder is not a prism? How a sphere is divided with finite no. of pyramids? Many problems were solved in examples so that the children understand different situations in which the surface areas and volumes are necessary for 3D objects.

Problems in the exercises were given related to daily life and which connects to apply previous knowledge. The academic standard prescribed for this chapter was taken in to consideration in setting up the problems. Problems relating to logical reasoning, communication, representation, connection besides problems were given through out the chapter to assess the children formatively.

Constructive approach was followed to make the children to understand various concepts in finding surface areas and volumes. For example to find the volume of cone, we begin with the objects in conical shape in the nature such as carrot, ice cream cone etc to give an idea of the shape. The formula to find the lateral surface area of cone i.e. $A = \pi r l$, Total surface area $A = \pi r(r+l)$ and volume of cone $V = 1/3^{\text{rd}} \pi r^2 h$ were derived by the basic concepts of area of circle, area of sector and volume of cylinder. The L.S.A of cone is explained by an activity and algorithm. T.S.A of cone is obtained by adding area of the base i.e. circle and L.S.A of cone. By comparison of a cylinder and a cone of same

radius and same height and through a simple experiment the volume of the cone is concluded as it will be $\frac{1}{3}$ rd volume of cylinder. Similarly the surface area and volume of sphere was also derived in a systematic way in which logical reasoning is involved.

Mensuration should always be learned through daily life situations. Every concept in these chapters is carefully taken, promoted the children participation for better understanding. Exercises like do this, try these besides think and discuss were provided to assess the children throughout the chapter to know the skills and application of knowledge in different situations.

The learning environment in the school must also give space to the whole range of mathematical processes- problem solving, use of heuristics, estimation and approximation, use of patterns, visualization, representation, reasoning and proof, making connections and mathematical communication.

- SCF 2011

H. MENSURATION-Approach to a chapter

◆ Introduction :

Observation and identification of 2D and 3D objects in groups

◆ Surface area of cuboid :

- ◆ To derive the formula to find the lateral surface area and total surface area
- ◆ Finding the lateral surface area and total surface through an activity

◆ Volume of cuboid :

- ◆ Explaining the concept through an activity.
 - ◆ Understanding the capacity of a vessel
 - ◆ Deriving the formula to find the volume of cuboid
 - ◆ Deriving the formula to find the volume of cube through volume of cuboid
 - ◆ Finding the volume of right prism by the volume of cuboid and cube.
 - ◆ Deriving the formula to find volume of pyramid by activity.
- ◆ Refer textbook for the presentation of the chapter which includes surface areas and volumes of cuboid, cube and pyramid

◆ Right circular cylinder :

- ◆ Concept of cylinder through an activity.
 - ◆ Identification of right circular cylinders and other cylinders.
 - ◆ Deriving the formula to find lateral surface area of cylinder-do this exercise
 - ◆ Deriving the formula to find total surface area of cylinder-do this exercise
 - ◆ Finding the formula to find volume of cylinder through an activity.
 - ◆ Understanding the concepts and application of knowledge through illustrative examples.
- ◆ (Refer the textbook exercises related to the above concepts)

◆ Right circular cone :

- ◆ Concept of cone through an activity
- ◆ Deriving the lateral surface area of cone by an activity.
- ◆ Deriving the total surface area of cone by an activity
- ◆ Deriving the volume of cone through the volume of cylinder.

◆ (refer textbook exercises related to the above topics)

◆ Sphere :

- ◆ Concept of sphere through the objects obtained in surroundings
- ◆ Deriving the formula of surface area of sphere through an activity.
- ◆ Finding the lateral surface area and total surface area of hemisphere.
- ◆ Deriving the volume of sphere through an activity.

◆ (refer textbook exercise related to the above topic)

◆ Revision

Revise the concepts of prism, cube, cuboid, cylinder, cone and sphere through What we have discussed.

The Mathematics that child learns to do inside and outside school remaining separate and disconnected. Of course, the larger issue here is of the relation between the school curriculum, and life outside school. Since Mathematics is an abstract branch of knowledge, one may think that there is little to be said about its connecting with culture and everyday life.

- SCF 2011

CHAPTER-6

Teacher Preparation

Teaching learning process is an important aspect of teaching a subject. It includes teacher preparation, presentation and evaluation. Now the curriculum and new text books provides an opportunity to modify the teaching learning process to reach each and every child of the class. Thus the teachers has to modify their teaching strategies in accordance with the prescribed academic standards at each level.

- ◆ Preparation is a vital stage to plan and visualise the teaching learning process in the class room.
- ◆ How to make children to participate in the teaching learning process is a very important aspect of preparation.
- ◆ Assessment of children through continuous comprehensive evaluation is another aspect of preparation .

THINK AND DISCUSS :

- ◆ Is the teaching learning process natural or artificial ?
- ◆ Is mathematics teaching taking place in accordance with nature of maths and nature of the child ?
- ◆ Do all the teachers know the changes taking place in teaching learning processes?
- ◆ Do the children are learning of maths only through teacher ?
- ◆ Do the children are learning mathematical concepts through their experiences ?
- ◆ What kind of changes do we expect from children in teaching maths ?
- ◆ Do the children applying their mathematical knowledge in solving problems in real life situations ? If not , why they are not able to apply ?
- ◆ Why people are saying that mathematical standards are decreasing in the days ? Is it a fault of the curriculum or teaching learning processes ?
- ◆ Is it a fault of children ?

The Andhra Pradesh state curriculum framework-2011 has reviewed all the above aspects and made recommendations in the revision of syllabus, suggestions to improve the teaching learning process and evaluation. In this context the syllabus is changed and new textbooks were prepared in accordance with national syllabus. Change is the order of the day. It is necessary that every teacher should understand the changes occurred in the society and process of teaching so as to make every child to participate in learning. So the teachers should revisit their teaching strategies and make necessary changes to improve the quality of mathematical education.

Changes takes place in text books : Let us go through what changes takes place in revising the textbooks once before how to change our teaching strategies.

The syllabus prescribed to 8th and 9th classes is divided in to various chapters keeping in the view to cover all areas. i.e.; arithmetic, algebra, geometry, mensuration, statistics and number system.

8th class	:	15 chapters
9th class	:	15 chapters.

**Children do not only
learn individually but
also in their
interactions with
other children
-SCF-2011**

Some features of new textbooks

- ◆ The chapters belonged to different areas were mixed together in the annual plan according to CCE
- ◆ Introduced every chapter by a discussion or a dialogue keeping in the mind of children's previous knowledge and daily life experiences.
- ◆ Every chapter is divided in to sub topics and encouraged children participation by discussion method.
- ◆ To assess the child at each sub topic a small exercise under the title "do this" is given. This will help the child too to assess himself on a particular topic.
- ◆ To encourage the children to think logically and give reason to his/her answer some questions under "try this" and "think and discuss" were given.
- ◆ Teaching learning material has an important role in teaching learning process. It will the teacher to motivate and for interaction.
- ◆ Most of the topics in the chapters are meant to encourage the children to discuss with their friends and in groups.
- ◆ The additional topics given in the new textbooks such as Highlight from history, Brain teasers, Do you know etc; will promote the children to develop positive attitude and interest towards learning of mathematics.

A teacher can see the greater shift in his/her teaching in the class using these new textbooks. So every teacher must go through the content included and method adopted in these new textbooks. Every teacher has to plan activities and concept mapping questions before every lesson.

Teacher should act as a facilitator and as a guide to encourage children to participate in the discussion and activities. Also encourage them to learn maths with joy.

THINK AND DISCUSS :

What kind of teaching learning process is required to adopt in the classroom with these new textbooks ? Compare with the process that you are following now.

Steps in teaching learning process

- ◆ Motivating the children with previous knowledge/ daily life experiences/ discussion with introduction given in the textbook.
- ◆ Make the children to understand the concepts by incidents/reading the examples/solving problems.
- ◆ Make them to participate in group discussions and activities for concluding the facts.
- ◆ Ask them to solve the problems in "do this" exercise individually and "try these" questions in groups.
- ◆ Provide situations to identify the errors committed by the children on their own and make them to correct mistakes.
- ◆ Assess the children's performance time to time and record in CCE register.
- ◆ Motivate the children to solve the problems given in the exercises on their own. Help them if necessary.
- ◆ Create new problems to encourage fast learners to solve and find new relations.

"Teaching learning process is a result of over all discussion takes place between two interrelated corresponding axioms"

-Bernand russels

Self reflections:

- ◆ Explain the steps of teaching a lesson from 8th or 9th class mathematics.
- ◆ Prepare a table of activities (individual/group/whole class) for a lesson from 8th or 9th class mathematics.

ROLE AND RESPONSIBILITY OF A TEACHER

- ◆ How a teacher is to be prepared before taking the class?
- ◆ What kind of material he/she has to prepare ?
- ◆ What kind of activities are to be designed to motivate all children to participate in learning process?
- ◆ What are the activities to be planned at the end of class room teaching?

Any kind of textbook is not an alternative to the classroom teacher. The teacher should keep the objectives (academic standards) in mind to plan the teaching learning process.

Let us understand some necessary steps and material to plan the process:

Teacher preparation

1. Preparation : Teaching :

- ◆ Should list out the academic standards of the lesson.
- ◆ Read the chapter thoroughly and solve the problems given in the exercises.
- ◆ Plan individual /group activities are to be given to the children.
- ◆ Plan activities for A, B, C group children as per the content.
- ◆ List out various daily life examples to connect with the topics.

2. Preparation:Teaching learning material :

- ◆ Teaching learning material mostly includes classroom interaction material such as charts, sketch pens, grid paper, graph paper .
- ◆ While teaching statistics, geometry, symmetry, required colourful pictures must be collected from magazines.

◆ Here some material required for every math teacher is listed.

- | | |
|--------------------------|-------------------------------------|
| 1. charts -white, colour | 9. colour papers |
| 2. geometry box | 10. scissor |
| 3. isometric sheets | 11. long scale(1 m) |
| 4. grid paper | 12. graph board |
| 5. graph paper | 13. mirror |
| 6. dice | 14. rope of uniform width |
| 7. coins | 15. 3d objects (cube, cuboid etc;) |
| 8. trace paper | 16. procircle. |

3. Preparation:Time management :

- ◆ Plan to children activities in the allotted time.
- ◆ See that net time is used to involve every child in learning.
- ◆ Plan the time for motivation, presentation and discussion, assessment
- ◆ See ample time given for problem solving.

4. RESPONSIBILITIES OF A TEACHER :

- ◆ Going to class in time and come back only after the allotted time.
- ◆ See all the children to participate in the process of teaching.

- ◆ Allotting additional activities or problems to intelligent children.
- ◆ Record the formative assessment records time to time.
- ◆ Writing teacher dairy and updating it by the inclusion of reflections.
- ◆ Checking the notebooks of the children to make them understand the mistakes they made.
- ◆ Follow formative assessment procedure to record the children progress.
- ◆ Prepare question papers keeping the academic standards in the mind.
- ◆ Plan the activities/more problem worksheets for backward children.
- ◆ Discuss the progress of the children with parents time to time.
- ◆ Discuss issues with colleagues other teachers to get more clarity in content and methodology.
- ◆ Go through reference books, internet for more conceptual clarity.

Self reflections :

How can i plan and prepare to teach the chapter "probability" for 9th class children?

Inadequate teacher preparation and support acts negatively on all of school Mathematics. Generally a teacher teaches how he learned from his teacher. Due to many pressures teachers adopt short cuts to achieve better marks and ranks. Learning achieved in this way is very short lived.

- APSCF-2011.

Chapter - 7

Continuous Comprehensive Evaluation - Understanding

Introduction

In order to identify 'How students are participating in learning process?', 'To what extent students are comprehending on concepts?', 'whether the teaching methods are successful in achieving the academic standards of not?' and to determine the goals of education 'Comprehensive Evaluation' is necessary. Now, in our schools, this is being carried out in two ways, 1. Formative Evaluation and 2. Summative Evaluation. In this chapter we discuss about the procedure of conducting evaluation and how to prepare question papers suitably.

- ♦ **What to be evaluated in Mathematics? How?**
- ♦ **What aspects must be taken care in Formative Evaluation?**
- ♦ **What aspects must be taken care in Summative Evaluation?**
- ♦ **How to identify and consider the mistakes done by the students? How to make use of this knowledge in teaching process?**
- ♦ **What are the principles to keep in mind while preparing a FA / SA question paper?**

What to be evaluated in Mathematics?

Mathematics teaching has the following objectives -

1. Children should be able to develop skill and understanding in the various curricular areas related to number and space.
2. Children should be able to reason mathematically.
3. Children should be able to pursue assumptions to their logical conclusion
4. Children should be able to handle abstraction.
5. Children should develop problem solving abilities- understand/formulate problems, develop a variety of strategies to solve them, verify and interpret results, generalize to new situations)
6. Children should develop confidence in using Mathematics meaningfully.

By consolidating all these, the main objective of evaluation in mathematics is to determine the following competencies.

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connection
5. Representation and Visualisation

Note : Clear statements about what students must know and be able to do are called '**Academic standards**'.

How to conduct Evaluation Process?

As a part of evaluation 'Formative and Summative Evaluations' should be conducted, simultaneously with teaching learning process, to measure the level of achievement of the child.

Formative Evaluation :

Formative Assessment is a assessment conducted during the process of teaching. Some of the main features of formative assessment are i. It is diagnostic and remedial and in the form oral, written and performance, ii. Enables teaches to adjust teaching to take account of the results of assessment, iii. Builds on student's prior knowledge and experience in designing what is taught and iv. Encourages students to understand the criteria that will be used to judge their work. The teacher observes and record the children progress and as well as learning gaps through the following.

(1) Participation and Reflections of the students during teaching learning process.(2) Class work, Homework, Portfolio's, Assignments etc., (3) Slip Test (4)Children Projects.

Formative Assessment is Assessment for Learning and assessment as learning. It is through observation of student responses, student engagement, student notebooks, assignments and other written works. Formative Assessments will be conducted by the teacher during instructions. We shall discuss the above tools in detail.

1. Participation and Reflections of the students :

Teacher must observe the students how they are participating in the learning process and how they respond to the explanations and discussions. Activities given in the book help to make children participate in learning process more actively. By instructing and guiding the students to attempt the exercises like 'Try these', 'Think and Discuss' in the class room itself they must be encouraged to understand the concepts by firsthand experience, to discuss in groups and to express their opinions to convince the others. Then they must be evaluated how far they understood the concepts.

2. Class work, Homework, Portfolio's, Assignments :

By correcting class work and home work note books, portfolios and assignments teacher must estimate the students that how they are able to solve problems, gather information, record the data in proper formats, apply the mathematical reasoning. Teachers must verify class work note books to find whether the students are able to solve 'Do these' exercise by their own, home work note books to find how students solve the exercise problems, projects/assignments to find how the students gather information, analyse and record.

3. Slip Test :

Slip test may be conducted then and there the teacher wants to. Prior intimation need not be given to students. This must be conducted in the instructional period only. It may be conducted to test one or two concepts, but with entirely new problems. This is meant for evaluating the competencies; Problem solving and Reasoning and proof.

4. Children Projects :

In teaching process, it is general practice to do activities and allotting projects to students. Projects may be allotted to students individually or in groups. For this students have to plan the activities and questioner of formats to execute in field level. They have to gather information, process it by discussion. Teachers must guide and observe the students throughout the project execution and assess the students by analyzing the reports which they submit.

Summative Assessment :

Summative assessment takes place after a period of instruction and requires making a judgment about the learning that has occurred. This is through using paper pencil tests with prior notice of the examination schedule, may be at the end of term / semester or academic year. The examination has to be conducted for 2½ hour and by keen verification of students' responses they must be assessed. The following procedure has to be followed for summative evaluation.

- ◆ Summative Assessment is a terminal tests in nature and shall be conducted twice in a year
 - o SA1 - in October syllabus upto October
 - o SA2 - in March/April syllabus upto October 30% to 40% and

Remaining syllabus
60% to 70%
- ◆ Teachers must prepare the subject wise question papers to test the prescribed academic standards.
- ◆ Question paper must be prepared on the basis of weightage tables of academic standards and subject prescribed.
- ◆ Question paper should contain all sorts of questions, such as i. descriptive, ii. Short answer, iii. Very short answer and iv. Objective type (Fill in the blanks, Multiple choice, Matching) questions.
- ◆ Oral test need not be conducted particularly for this, teacher may award marks for this section on the basis of his / her observations in the previous formative test or by present observation on the students as per the academic standards prescribed.

Formative Evaluation assess the childrens' learning through observation, oral tests and slip tests, whereas the Summative Evaluation assess the childrens' level of learning through written test only. But in lower classes (I and II classes) oral test also be given whightage.

Whatever may be the evaluation, ultimate aim is to achieve the academic standards among the children. Accordingly activities, debates and discussions must be conducted and children must be assessed.

Principles to be noted while conducting Formative and Summative Evaluations:

- ◆ Previously students are evaluated only on the basis of written tests, that to with knowledge based questions. But in present students are to be evaluated in many ways of learning.
 - 1) Participation and Reflections of the students during teaching learning process.
 - 2) Class work, Homework, Portfolio's, Assignments etc.,
 - 3) Slip Test
 - 4) Children Projects.
 - 5) Written test based on Academic standards.

First four of the above are tools for Formative Evaluation, where as 4th and 5th are the tools for Summative Evaluation.

- ◆ CCE refers to a system of school based assessment that covers all aspects of students growth and development. Evaluation of student's growth and development is continuous process rather than an event, built into the total teaching learning process and spread over the entire span of academic session. For the entire process the teacher is more important. Teacher has to plan execute the teaching learning process according to academic standards and simultaneously evaluate the students with teacher made question papers.
- ◆ Generally question papers constitute of questions mainly to test the knowledge from the text book. Now academic standards are fixed to every subject and class and new teaching learning procedures are suggested to achieve those academic standards. So the evaluation should also aim at achieving the academic standards.
- ◆ Questions in tests or projects must provoke the students to think, explore, compare and contrast between the other subjects and daily life and encourage them to express their own experiences and views.
- ◆ Problem solving: Under this academic standard questions of verbal and non verbal, data comprehension, graphs, analysis and synthesis, constructions, etc. with complexity shall be asked. The complexity of a problem is depends on making connections among the various concepts of subjects and real life situations, number of steps, number of operations, context unravelling and nature of procedures.

- ◆ Reasoning and Proof: Under this academic standard questions must aim at reasoning between various steps, understanding and making mathematical generalizations and conjectures, understanding and justifying procedures, examining logical arguments, understanding the notion of proof, use inductive and deductive logic.
- ◆ Communication: Under this academic standard questions must evaluate skills of writing, reading and expressing mathematical notations (verbal and symbolic forms), Creating mathematical expressions, Explaining mathematical ideas in her own words like- a square is closed figure having four equal sides and all equal angles, Explaining mathematical logic.
- ◆ Connections: Under this academic standard questions must comprise Connecting concepts within a mathematical domain, Connecting mathematics to different subjects and with daily life, connecting concepts of different mathematical domains like data handling and arithmetic or arithmetic and space, connecting concepts to multiple procedures.
- ◆ Visualization and Representation: Under this academic standard questions must test interpretation and reading data in a table, number line, pictograph, bar graph, 2-D figures, 3-D figures, pictures, Making tables, number line, pictograph, bar graph, pictures and Making use of mathematical symbols and figures.
- ◆ While conducting Formative or Summative evaluations for different classes the following academic standard wise weightages are to be followed scrupulously.

Class	Description	Formative Evaluation					P	Summative Evaluation										
		Participation & Reflections	Note books	Projects	Slip tests	Total		Problem solving		Reasoning and Proof		Communication		Connections		Visualization Representation		Total
1-2	Weightage	20%	20%	20%	40%	100%		Oral	Written	Oral	Written	Oral	Written	Oral	Written	Oral	Written	
	Marks	10	10	10	20	50M		10%	40%	10%	-	-	10%	10%	-	10%	10%	100%
3-5	Weightage	20%	20%	20%	40%	100%		50%		20%		10%		10%		10%		100%
	Marks	10	10	10	20	50M		25		10		5		5		5		50M
6-9	Weightage	20%	20%	20%	40%	100%		40%		20%		15%		10%		15%		100%
	Marks	10	10	10	20	50M		40		20		15		10		15		100M

Principle to be followed while writing a question paper:

- ♦ Teacher must prepare question paper by him / herself for Summative Evaluation and for slip test in Formative Evaluation.
- ♦ Slip test in Formative Evaluation may be conducted once or in set of small test, totally for 20 marks.
- ♦ Question paper of Summative Evaluation should be prepared with different kinds of questions as per the following weightage table.

S. No	Type of Question	1 to 5 classes			6 to 9 classes		
		No of Questions	Marks for each	Total	No of Questions	Marks for each	Total
1	Essay	4	5	20	4	10	40
2	Short type	8	$2\frac{1}{2}$	20	8	5	40
3	Very Short type	5	1	5	10	1	10
4	Objective	5	1	5	20	$\frac{1}{2}$	10

Note: Objective type includes fill in blanks, multiple choice, true or false (for 1-5) and matching questions.

Academic standard wise weightage table for Summative Evaluation

Class	Description	Problem solving		Reasoning and Proof		Communication		Connections		Visualization Representation		Total
		Oral	Written	Oral	Written	Oral	Written	Oral	Written	Oral	Written	
1, 2	Weightage	10%	40%	10%	-	-	10%	10%	-	10%	10%	100%
	Marks	5	20	5	-	-	5	5	-	5	5	50
3, 4, 5	Weightage		50%		20%		10%		10%		10%	100%
	Marks		25		10		5		5		5	50
6 to 9	Weightage		40%		20%		15%		10%		15%	100%
	Marks		40		20		15		10		15	100

- ◆ Simultaneously the teacher must follow the academic standard wise weightages as follows. "
In summative evaluation question paper weightage for problem solving must be followed and weightages for the other academic standards may be altered according to the content to be tested.
- ◆ **Essay questions** must provoke the students to think, explore, compare and contrast between the other subjects and daily life and encourage them to express their own experiences and views. These questions must require descriptive answers. Theorems, constructions, solving equations and graphs may be included.
- ◆ **Short questions** must seek answers about one concept, figure, problem with 3 or 4 steps, little explanations only.
- ◆ **Very short answer** questions must be of problems with 1 or 2 steps, oral, definitions, formulae, symbols interpretation from tables or graphs, etc.
- ◆ Objective type questions must very very short and can be calculated in no time. These question must not be with multiple answers.
- ◆ All questions must be new (not from the text book) to the possible extent.
- ◆ In every academic standard essay type question must be given internal choice (one out of two should be answered).
- ◆ In any chapter if large questions are not available, two questions (a and b) may be given in each essay question.

How to consider the mistakes written by the students?

Continuous Comprehensive Evaluation is not a simple written test, but it refers to a system of school based assessment that covers all aspects of student's growth and development such as physical and mental development, social behavior, attitudes and interests. It includes both scholastic and co-scholastic areas and should be comprehensive in nature. This is in line with the goals of education. CCE is carried out simultaneously with the teaching learning process, i.e., in class room, laboratory, library, play ground, projects and other activities

During the teaching learning process when a student does mistakes in giving oral answers or representing in writing we don't point out much, instead we use to teach the same concept again in alternative mode and ask the student to repeat the study. In correcting note books or evaluating examination answer scripts it is not enough to correct the mistake(what we think), but we should enquire what is his / her idea in answering so. By this we can understand his draw backs such as i. Unable to understand language, ii. Don't know the concepts, iii. Misunderstanding the instructions and iv. Unable to analyse the problem. Then the student may be given proper guidance to learn.

Due to these reasons teachers must be cautious in using the terms, symbols, words and phases during teaching and writing the question paper. Instructions and questions must be in simple sentences.

Students are unable to use their understanding in written tests. Why?

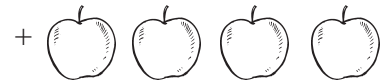
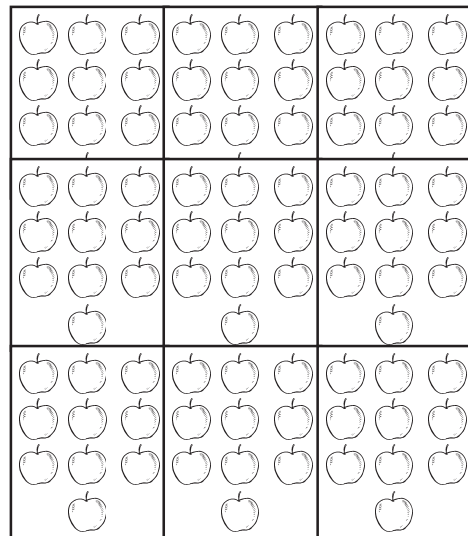
Generally students are able to use the mathematical concepts, what they comprehend in class room, in their daily life effectively. But they are unable to use the same competencies in writing a test. This is because the difficulty with correlation of mathematics with symbols, language writing procedures. In order to overcome this, pin pointed explanation, enough exercises and drilling must be given to them. Introduction mathematical terms and writing procedures must be explained with due care.

Consider an example. A student answered an addition problem as given aside.

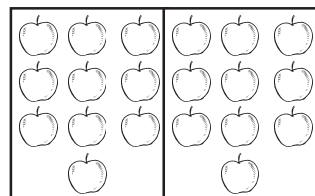
$$\begin{array}{r} 94 \\ 28 \\ \hline 1112 \end{array}$$

Instead of giving the correct answer 122, the teacher must analyse the students understanding. The student is unable to identify the place values. If it is identified and proper reinforcement with concrete examples is given then the student does not do the same mistake again.

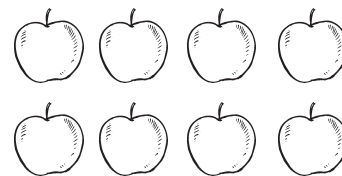
94 =



28 =



+



94 Apples = 90 Apples + 4 Apples

28 Apples = 20 Apples + 8 Apples

= 110 Apples + 12 Apples

= 110 Apples + 10 Apples + 2 Apples

= 120 Apples + 2 Apples

Total Apples = 122 Apples

If it is explained as "Here sum of units places $4 + 8 = 12$ has one 10 and 2. This 10 has to be added to tens place", then students will definitely understand the concept.

Chapter - 8

Self Evaluation Sheet

Teacher's name : _____ Qualification : _____
 Designation : _____ School : _____
 Mandal : _____ District : _____

From your experience in the training program express your opinion for following questions.

	Extensive	Better	Satisfactory
1) Training has been given according to new text books.	()	()	()
2) Key principles of syllabus reflected in textual lessons.	()	()	()
3) Arrangement of chapters is scientific and gave equal importance to all the chapters.	()	()	()
4) Language and print in text books are appropriate to the students.	()	()	()
5) Understood the method of analyzing each chapter.	()	()	()
6) Different items incorporated in chapters are suitable to evaluate all academic standards.	()	()	()
7) Understood the method of teaching verbal problems.	()	()	()
8) Learnt what is probability and its scope.	()	()	()
9) Identified the need for Proofs in Mathematics.	()	()	()
10) Discussions were conducted about the approach papers on different areas of mathematics.	()	()	()
11) Understood the importance of irrational numbers and method of marking them on number line.	()	()	()
12) Learnt about the variable, equation and linear equation in algebra.	()	()	()
13) Understood the axiomatic approach to comprehend the geometry.	()	()	()
14) Trained to construct geometrical constructions properly.	()	()	()
15) Enhanced the understanding of different statistical methods and graphical representations.	()	()	()

- | | | | | |
|-----|---|--------|--------|--------|
| 16) | Carried out the activities to understand areas and volumes. | () | () | () |
| 17) | Oriented to get prepared for teaching mathematics in new methods. | () | () | () |
| 18) | Knew the need for creating additional questions. | () | () | () |
| 19) | Discussed to evaluate each and every student as per CCE. | () | () | () |
| 20) | Training helped me to prepare question papers on the basis of academic standards. | () | () | () |

- 1) Which session impressed you more? : _____
- Why it impressed you? : _____
- 2) Which session did not satisfy your needs? : _____
- Why do you think so? : _____

Date

Signature of the participant